On the limits of macroprudential policy

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Abstract

This paper studies how macroprudential policy tools can complement the interest rate-based monetary policy in achieving a selection of dual stabilization objectives. We show analytically in a canonical New Keynesian model with collateral constraints that using the loan-to-value ratio as an additional policy instrument does not resolve the inflation-output volatility tradeoff. Perfect targeting of inflation and either credit or house prices with monetary and macroprudential policy is possible only if the role of credit in the economy is sufficiently small. Any of these three dual stabilization objectives can be achieved with the monetary-fiscal policy mix. The identified limits to the LTV ratio-based policy are related to its predominantly intertemporal effect on decisions made by financially constrained agents.

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1 Introduction

It is well understood that not all business cycle fluctuations are efficient and hence an appropriately concocted set of policy measures that are aimed at limiting volatility of selected macrocategories can improve social welfare. Consistently with this view, most of contemporary macroeconomic models that are used to inform policymakers assign an important role to stabilization policies. At the same time, it is clear that there are limits to what such policies can achieve. One of them is related to the fact that the number of available instruments is usually smaller than the number of targets that one might want to hit. This violates the famous Tinbergen rule that postulates the need for at least as many independent instruments as the number of policy goals.¹ In consequence, policymakers have to resolve tradeoffs between various stabilization objectives.²

This type of dilemmas also applies to monetary policy, which is widely considered the preferred policy to stabilize business cycles, also because of its not being subject to big implementation lags. According to the standard New Keynesian model, it is in general not possible to perfectly stabilize both inflation and the output gap using just the short-term interest rate (Gertler et al., 1999; Woodford, 2003), a result that is often referred to as the inflation-output volatility tradeoff. Naturally, the policy dilemmas become even more difficult to resolve in the presence of financial frictions as these make additional stabilization objectives relevant, or if the central bank is explicitly responsible for maintaining financial stability.³

Following the recent financial crisis, a new type of policy, dubbed macroprudential, has been proposed as a promising way of handling imbalances that arise from financial market imperfections. This policy is based on a set of tools typically used by the financial supervisory authorities at a microeconomic level, and applies them to limit vulnerabilities of the whole financial system.⁴ Various macroprudential policy instruments have been proposed by the standard-setting or policy-making institutions like the Bank for International Settlements or International Monetary Fund, some of them have also been used by central banks in various countries (IMF, 2011). Many of these tools are aimed to operate at business cycle frequencies. In particular, countercyclical use of capital buffers was advocated by the Basel Committee on Banking Supervision in its Basel III recommendation to smooth the credit cycle, i.e. prevent excessive growth of credit during booms and ensure its availability during

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¹As demonstrated by the literature on instrument instability, meeting the Tinbergen rule is not sufficient for the stabilization policy to be implementable, see e.g. Holbrook (1972) or Lane (1984).

²There is a long list of papers discussing how these tradeoffs should be resolved. In the context of a standard monetary-fiscal policy mix, see e.g. Dixit and Lambertini (2003), Schmitt-Grohe and Uribe (2004) or Siu (2004).

³There is extensive literature on whether and how monetary policy should respond to asset prices to reduce the cost of a possible financial crisis. See e.g. Bernanke and Gertler (2001), Carlstrom and Fuerst (2007) or Faia and Monacelli (2007).

⁴See Clement (2010) for the origins and evolution of macroprudential policy.
periods of financial stress (BCBS, 2010). As evidenced by the experience of some economies, in particular Hong Kong and South Korea, adjustments in the loan to value (LTV) ratio can serve a similar purpose (Hong Kong Monetary Authority, 2011; Kim, 2014).

A natural question that emerges in this context is: How efficient can these new policy tools be in meeting particular stabilization objectives, and how do they interact with monetary policy? A number of papers have looked into this issue and the literature is growing fast. The findings are usually moderately positive as they suggest that macroprudential policy instruments are quite efficient in stabilizing the financial sector and hence can be considered a useful complement to monetary policy, see Galati and Moessner (2013) for a survey. However, there may be important interactions between the two policies that justify the need to coordinate them (De Paoli and Paustian, 2013; Bodenstein et al., 2014; Cecchetti and Kohler, 2014). This literature also includes papers that use dynamic stochastic general equilibrium (DSGE) models with sticky prices and financial frictions, in which the central bank fully controls the short-term interest rate while the macroprudential authority sets the LTV or bank capital adequacy ratio.\footnote{See e.g. Darracq-Pariés et al. (2011), Angeloni and Faia (2013) or Lambertini et al. (2013). Quint and Rabanal (2014) and Brzoza-Brzezina et al. (2015) look at this issue in the context of a heterogeneous monetary union. Brzoza-Brzezina et al. (2014) show how macroprudential policy may be limited by the presence of multi-period loans and occasionally binding collateral constraints.} They offer conclusions that are based on stochastic simulations for various specifications of the feedback rules that describe the decisions made by the monetary and macroprudential authorities.

This paper adds to our understanding of how macroprudential policy can contribute to macroeconomic stability by taking a different and more analytical perspective. To this end, we consider a simple New Keynesian framework augmented with housing and collateral constraints faced by borrowers as in Iacoviello (2005), and define the LTV ratio as an additional policy instrument. This framework is then used to propose and prove several important qualitative statements about the ability of monetary and macroprudential policy to jointly meet a selection of dual stabilization objectives, where stabilization is understood as eliminating deviations of key macrovariables, such as inflation, output, credit and house prices, from any exogenously given targets. In particular, these targets can vary over time with shocks hitting the economy and also approximate arbitrarily closely the goals derived from the welfare-based loss function.

As regards the methodology, we use a linear approximation to the model equilibrium conditions. This yields the system of equations that is tractable enough to allow us to prove the key propositions analytically. Our analytical approach can be contrasted with the previous DSGE literature that is based on numerical simulations with larger models, for which qualitative results cannot be derived and hence one might be concerned whether the reported findings go through for alternative and plausible calibration choices. The local flavor of the framework used in this paper also makes it different from the recent research
that employs global methods to study macroprudential policy in the presence of occasionally binding constraints (e.g. Jeanne and Korinek, 2010; Benigno et al., 2013; Brunnermeier and Sannikov, 2014).

The main findings obtained with our benchmark New Keynesian macro-financial model can be summarized as follows. First, there does not exist a stable equilibrium in which inflation and output are kept at their targets by an appropriate mix of monetary and macroprudential actions. In other words, having the LTV ratio as an additional instrument to the short-term interest rate controlled by the central bank does not resolve the standard inflation-output stabilization tradeoff. In this respect, macroprudential policy is less efficient than government spending policy, which, if combined with monetary policy, can achieve this dual objective. Second, using monetary and macroprudential policy to simultaneously target inflation and credit is possible only if the role of the latter in the economy is sufficiently small. Again, government spending is a more efficient alternative to the LTV-based policy as applying it together with monetary policy such that inflation and credit are at their targets always leads to a stable equilibrium. Third, similar conclusions hold for the ability of monetary and macroprudential (or fiscal) policy to stabilize inflation and house prices.

The following intuition behind these identified failures of the monetary-macroprudential policy mix can be offered. Monetary policy affects patient households’ intertemporal choices by making their current consumption more expensive or cheaper relative to that in the future. Similarly, LTV-based macroprudential policy also operates at an intertemporal margin as it affects impatient households’ access to credit and hence their ability to smooth consumption over time. However, adjustments in the short-term interest rates also significantly influence the financial position of financially constrained borrowers through their impact on the cost of credit. Since impatient households’ ability to absorb shocks is limited, the monetary policy-induced changes in their disposable income strongly affect their spending and labor supply. By working mainly intertemporally, macroprudential policy is not very efficient at offsetting such intratemporal spillovers whenever the chosen stabilization objectives require it to do so. As a result, sticking to its goals may be consistent only with explosive paths of the LTV ratio and some of the macrovariables.

In contrast, government spending operates mainly intratemporally as it affects disposable income of the taxpayers and hence it is a much better complement to monetary policy. Therefore, our analysis suggests that for the monetary-macroprudential policy mix to be efficient, its latter component should be based on instruments that, unlike the LTV ratio, have a sufficiently strong intratemporal effect. These instruments should be able to influence the balance sheets of agents, and not only distribution of their choices over time.

The rest of this paper is organized as follows. Section two presents the benchmark New Keynesian macrofinancial model and its log-linearized version. Section three establishes and discusses our main results. Section four concludes. A full list of model equations, as well as
proofs and more details of all propositions stated in the text are presented in the Appendix.

2 Benchmark model

Our benchmark model is a standard New Keynesian (NK) setup, extended to include housing and financially constrained agents that can borrow only against housing collateral as in Iacoviello (2005). There are three authorities that fully control three policy instruments: the central bank sets the short-term interest rate, the government determines its purchase of final goods, and the macroprudential authority controls the loan-to-value (LTV) ratio.

The rest of this section briefly describes the problems facing the agents and a log-linearized version of the model. A full list of equations making up the original model is given in the Appendix.

2.1 Theoretical framework

2.1.1 Households

There are two types of households whose preferences differ in the degree to which they discount the future utility flows. This makes impatient households natural borrowers, and the patient ones natural lenders. We denote these two types by $I$ and $P$, respectively, and the size of the former by $\omega$, with the total measure of households normalized to unity. Within each group $i = \{I, P\}$, a representative agent maximizes

$$
E_0 \left\{ \sum_{t=0}^{\infty} \beta_t^t \left[ \log c_{i,t} + A \nu_t(\chi_{i,t}) - \frac{n_{i,t}^{1+\varphi}}{1 + \varphi} \right] \right\}
$$

where $A > 0$, $0 < \beta_I < \beta_P < 1$, $c_t$ is consumption, $n_t$ is labor supply, and $\nu_t$ is a function describing how housing stock $\chi_t$ affects the utility.

Patient households’ maximization is subject to a standard budget constraint

$$
P_t c_{P,t} + P_{\chi,t}(\chi_{P,t} - \chi_{P,t-1}) + D_t + T_t \leq W_t n_{P,t} + R_{t-1} D_{t-1} + \Xi_t
$$

where $P_t$ is the price of final goods, $P_{\chi,t}$ denotes the price of housing, $W_t$ is nominal wage, $\Xi_t$ denotes profits from monopolistically competitive firms, $T_t$ is lump-sum taxes, while $D_t$ stands for deposits paying risk-free rate $R_t$.

Impatient households’ budget constraint is similar, except that these agents do not own firms nor pay taxes, and the interest charged on their loans $L_t$ is also given by $R_t$

$$
P_t c_{I,t} + P_{\chi,t}(\chi_{I,t} - \chi_{I,t-1}) + R_{t-1} L_{t-1} \leq L_t + W_t n_{I,t}
$$
Additionally, impatient households’ optimization is subject to the collateral constraint

\[ R_t L_t \leq m_t P_{\chi,t} \chi_{I,t} \] (4)

where \( m_t \) denotes the LTV ratio, the steady-state of which is assumed to equal unity.

### 2.1.2 Firms

Final output \( y_t \) is produced by perfectly competitive firms that aggregate intermediate goods indexed by \( \nu \) according to

\[ y_t = \left[ \int_0^1 y_t(\nu) \frac{1}{\mu} d\nu \right]^\mu \] (5)

where \( \mu > 1 \).

Intermediate goods producing firms operate in a monopolistically competitive environment and use a production function

\[ y_t(\nu) = \varepsilon_t [\omega n_{I,t}(\nu) + (1 - \omega) n_{P,t}(\nu)] \] (6)

where \( \varepsilon_t \) is exogenous productivity. These firms set their prices according to the Calvo scheme, with the probability of not receiving the price change signal given by \( \theta \).

### 2.1.3 Market clearing

We impose a standard set of market clearing conditions. In particular, we assume that the stock of housing is fixed at \( \chi \) and hence the housing market clearing condition can be written as

\[ \chi = \omega \chi_{I,t} + (1 - \omega) \chi_{P,t} \] (7)

Equilibrium in the labor market implies (for \( i = \{I, P\} \))

\[ n_{i,t} = \int_0^1 n_{i,t}(\nu) d\nu \] (8)

Finally, the aggregate resource constraint is

\[ y_t = \omega c_{I,t} + (1 - \omega) c_{P,t} + g_t \] (9)

where \( g_t \) is government spending that is fully financed by lump sum taxes levied on patient households so that \( P_t g_t = T_t \).
2.1.4 Functional forms

The functional form for impatient agents’ housing utility is chosen to be \( \nu_I(\chi_{I,t}) \equiv \log \chi_{I,t} \). As regards that for patient households, we follow Justiniano et al. (2015) and assume that their preferences imply a rigid demand for housing at some fixed value \( \chi_P \). Such an asymmetric modeling of utility implies that there is no reallocation of houses across the two types of agents and that they are effectively priced by leveraged households, which is consistent with Geanakoplos (2010). This simplifying assumption also makes the analytical derivations of our results more tractable. However, as a robustness check, we will also check how our main findings change if we assume symmetric preferences, i.e. \( \nu_P(\chi_{P,t}) \equiv \log \chi_{P,t} \).

2.2 Log-linearized model

A log-linearized version of our benchmark model is given by ten equations that we list below. In what follows, variables without time subscripts denote their steady state values while a hat over a variable indicates its log-deviation from the non-stochastic steady state.

The standard consumption Euler equation for patient households is given by

\[
\hat{c}_{P,t} = \mathbb{E}_t \{ \hat{c}_{P,t+1} \} - \hat{R}_t + \mathbb{E}_t \{ \hat{\pi}_{t+1} \} \tag{10}
\]

while that describing intertemporal consumption choices of impatient households

\[
\hat{c}_{I,t} = \frac{\beta_I}{\beta_P} \mathbb{E}_t \{ \hat{c}_{I,t+1} \} - \hat{R}_t + \frac{\beta_I}{\beta_P} \mathbb{E}_t \{ \hat{\pi}_{t+1} \} - \left( 1 - \frac{\beta_I}{\beta_P} \right) \hat{\Theta}_t \tag{11}
\]

where \( \pi_t \equiv P_t/P_{t-1} \) is the (gross) inflation rate, assumed equal to unity in the steady state, and \( \Theta_t \) is the Lagrange multiplier on the collateral constraint (4). As can be seen, the difference in the consumption Euler equations across both types of agents results from their different discounting factors.

The log-linearized version of impatient households’ budget constraint (3) can be written as

\[
\frac{c_I}{\hat{l}} \hat{c}_{I,t} + \frac{1}{\hat{\beta}_P}(\hat{R}_{t-1} - \hat{\pi}_t + \hat{l}_{t-1}) = \frac{wn_I}{l}(\hat{w}_t + \hat{n}_{I,t}) + \hat{l}_t \tag{12}
\]

while the collateral constraint becomes\(^6\)

\[
\hat{R}_t + \hat{l}_t = \hat{m}_t + \hat{p}_{\chi,t} \tag{13}
\]

\(^6\)Since \( \beta_I < \beta_P \), the collateral constraint is binding in (and sufficiently close to) the non-stochastic steady state, which allows us to write it as an equality.
House prices are determined by the following housing Euler equation

\[(1 - \beta P + \beta I)\hat{p}_{X,t} = \beta_I E_t \{\hat{p}_{X,t+1}\} + \hat{c}_{I,t} - \beta_I E_t \{\hat{c}_{I,t+1}\} + (\beta_P - \beta_I)(\hat{\Theta}_t + \hat{m}_t)\]  

\[\tag{14}\]

and hence depend positively on the tightness of the collateral constraint.

Optimal labor supply schedules are given by

\[\hat{w}_t - \hat{c}_{P,t} = \varphi \hat{n}_{P,t}\] \[\tag{15}\]

\[\hat{w}_t - \hat{c}_{I,t} = \varphi \hat{n}_{I,t}\] \[\tag{16}\]

and postulate equalization of the real wage with the marginal rate of substitution between consumption and leisure.

Solving intermediate good firms’ problem gives the standard Phillips curve

\[\hat{\pi}_t = \beta_P E_t \{\hat{\pi}_{t+1}\} + \frac{(1 - \theta)(1 - \beta_P \theta)}{\theta}(\hat{w}_t - \hat{\varepsilon}_t)\] \[\tag{17}\]

and the log-linearized version of the aggregate production function can be written as

\[\hat{y}_t = \hat{\varepsilon}_t + \omega_n \hat{n}_{I,t} + (1 - \omega_n)\hat{n}_{P,t}\] \[\tag{18}\]

where \(\omega_n \equiv \omega n_t/n\).

Log-linearizing the goods market clearing condition (9) yields

\[\hat{y}_t = \omega_c \hat{c}_{I,t} + (1 - \omega_c - \omega_g)\hat{c}_{P,t} + \omega_g \hat{g}_t\] \[\tag{19}\]

where \(\omega_c \equiv \omega c_t/y\) and \(\omega_g \equiv g/y\).

Finally, rather than writing explicit rules for the three instruments controlled by the government, i.e. the interest rate \(\hat{R}_t\), the LTV ratio \(\hat{m}_t\) and government spending \(\hat{g}_t\), we describe them in terms of the targets that they are aimed to hit. Whenever a given policy is inactive, i.e. it is not assigned with any stabilization objective, its instrument is fixed at its steady state level.

Equations (10) to (19) together with three implicit policy rules determine the first-order accurate equilibrium evolution of 13 variables \(\{\hat{c}_{P,t}, \hat{c}_{I,t}, \hat{n}_{P,t}, \hat{n}_{I,t}, \hat{l}_t, \hat{w}_t, \hat{p}_{X,t}, \hat{\Theta}_t, \hat{\pi}_t, \hat{R}_t, \hat{m}_t, \hat{g}_t\}\) \(t=1\) to \(n\) given initial conditions \(\hat{l}_0\) and \(\hat{R}_0\) and for given exogenous sequence \(\{\hat{\varepsilon}_t\}_{t=1}^n\). As we will argue below, even though the model economy is driven only by shocks to productivity, our findings for the ability of various policy mixes to achieve certain stabilization objectives depend on dynamic properties of the system and not on the sources of aggregate fluctuations. Hence, restricting our attention to productivity shocks and using them to provide intuition for our main results should not be seen as restrictive.
2.3 Calibration

Wherever possible, the main results of this paper are derived analytically for all admissible parameter values. However, to illustrate and provide intuition for some of the findings, we also offer an impulse response analysis. In this section we discuss a benchmark calibration that underlies this part of the discussion and that is based on the US macroeconomic data.

We set the discount factor of patient households $\beta_P$ to 0.995 to match the average annual real interest rate of 2%. Following Campbell and Hercowitz (2009), the relative impatience of borrowers is calibrated at 0.5%, which implies $\beta_I = 0.99$. The inverse of the Frisch elasticity of labor supply $\varphi$ is set to a conventional value of 2. The relative share of impatient households $\omega$ and housing weight in utility $A_\chi$ are fixed at 0.25 and 0.045, respectively, to match the credit-to-GDP ratio of 0.45 and housing-to-GDP ratio of 1.2. We also assume a standard value for the price markup ($\mu = 1.2$) and the Calvo probability ($\theta = 0.75$). The share of government spending in output is set at its long-run average of 16.5%.

3 Results

We use the benchmark macro-financial NK model laid down in the previous sections to check if macroprudential policy, by using the LTV ratio as its instrument, can complement monetary policy in resolving the standard inflation-output volatility tradeoff or stabilize the financial sector without jeopardizing the central bank’s price stability objective. Ideally, to achieve this goal, one would like to derive the welfare-based loss measure in the spirit of Benigno and Woodford (2012) as this would allow to express the optimal policy in terms of explicit targets that a benevolent policy maker wants to hit. Unfortunately, our benchmark model is not well suited for such an analysis. This is mainly because heterogeneity in the discount factors makes the second-order approximation to patient and impatient households’ utility intractable and the definition of aggregate welfare problematic.\footnotemark

For this reason, we propose a different approach that can be characterized as follows. Throughout all of our analysis, we assume that the monetary authority always sets the short-term interest rate such that inflation is constant at all times. Such a policy eliminates the price dispersion cost related to price stickiness and hence is consistent with one of the social planner’s objectives. As we discuss later, this constant inflation target can be also generalized

\footnotetext{In particular, if the social welfare is defined as the (population weighted) average of patient and impatient agents’ utilities, it might be optimal for the planner to increase the LTV ratio to infinity, which would violate the microfoundations underlying the collateral constraint elaborated in Kiyotaki and Moore (1997). Cúrdia and Woodford (2009) and Benigno et al. (2014) circumvent the aggregation problems arising from heterogeneity in agents’ preferences by considering the limit in which the discount factors of the two types of households are equal. However, this assumption implies that LTV-based macroprudential policy does not affect the economy in the vicinity of the steady state as the collateral constraint is not binding. To see it, note that if $\beta_I = \beta_P$, the Lagrange multiplier on the collateral constraint $\Theta_t$ drops from the equations describing impatient households’ decisions (11) and (14).}
to a time-varying one that fluctuates exogenously. Next, we analyze if there exist a stable equilibrium in which appropriate adjustments in the LTV ratio allow to additionally stabilize either output, credit or house prices, where stabilization is understood as elimination of deviations from any exogenous (and hence possibly time-varying) targets. Since we do not need to impose any restrictions on how these targets depend on current or past shocks, they may also include those that are arbitrarily close approximations to the goals derived from the (unknown) welfare-based loss function. Note that the targets can also be set to zero, in which case we obtain a close characterization of the dilemmas considered by policy makers in real life, and which may be not necessarily consistent with normative implications of the underlying model structure. While presenting the outcomes under the monetary-macroprudential policy mix, we contrast it with a more traditional one, in which monetary policy actions are complemented with adjustments in fiscal policy that uses government spending as its instrument.

The main attraction of our approach is its tractability as it allows us to derive most of our important results analytically and hence complement the existing related DSGE literature that has resorted to numerical simulations. By providing intuition for our key qualitative statements, we can also better understand to what extent LTV-based macroprudential policy should be considered a good complement to standard monetary policy.

Before we proceed, it is instructive to examine what strict inflation targeting actually implies in our setup. By combining the production function (18) with market clearing condition (19) and substituting for labor inputs using the optimal labor supply schedules (15) and (16), we can derive the following equation describing the evolution of real wage

$$\hat{w}_t = -\varphi \hat{\epsilon}_t + (\omega_n + \varphi \omega_c) \hat{c}_{I,t} + [(1 - \omega_n) + \varphi (1 - \omega_c - \omega_g)] \hat{c}_{P,t} + \varphi \omega_g \hat{g}_t$$

(20)

According to the Phillips curve (17), for inflation to be constant at all times, wages must move one-to-one with productivity, i.e. the right hand side of the equation derived above must be equal to $\hat{\epsilon}_t$. This means that meeting the price stability objective after a positive technology shock requires an increase in an appropriately weighted sum of (log) aggregate demand components, where the weights on consumption of both types of households depend positively on their relative shares in output and labor supply. This relationship will be key to understand how our model economy responds to different variants of stabilization policy.

8Limiting fluctuations in the output gap is a standard prescription of the New Keynesian setup. There is also large literature postulating policy response to credit or asset prices, see e.g. Christiano et al. (2010), Lambertini et al. (2013) or Notarpietro and Siviero (2015).
3.1 Inflation-output volatility tradeoff

We start with the classical macroeconomic policy tradeoff, which concerns simultaneous targeting of inflation and economic activity. In general, and in line with the Tinbergen rule, the central bank cannot achieve both of these objectives using only one single instrument which is the short-term interest rate. However, one could hope that adding the LTV ratio as an additional policy tool will allow to hit both targets at the same time. This turns out not to be the case, which we formally state in form of the following proposition.

**Proposition 1.** In the benchmark macro-financial NK model, there does not exist a stable equilibrium in which monetary and macroprudential policy achieve full stabilization of inflation and the output gap.

More specifically, as we prove in the Appendix, a coordinated attempt of the central bank and macroprudential authority to perfectly stabilize both inflation and output at any exogenously specified targets necessarily leads to explosion in credit.

To see why it happens, suppose that our model economy is initially in the steady state equilibrium and then is hit by a positive shock to productivity $\hat{\epsilon}_t$ that lasts only one period. Figure 1 shows the dynamic responses to this scenario in a model that is calibrated as discussed in the previous section, assuming that the short-term interest rate and LTV ratio are set such that inflation and output are constant at all times.\textsuperscript{9} Note that the zero target for output deviations from the steady state is assumed only to facilitate the exposition and the logic of our argument is the same if this target is replaced with any function of current and past productivity shocks.

To understand the responses, recall that inflation remains unchanged only if wages go up exactly by as much as productivity. Using this observation with equation (20) and the market clearing condition (19) for constant output results in the following restriction

$$\omega_n \hat{c}_{I,t} + (1 - \omega_n) \hat{c}_{P,t} = (1 + \varphi) \hat{\epsilon}_t$$

which means that the weighted average of (log) consumption of patient and impatient households must go up for inflation to be constant. Note that, with passive fiscal policy (no change in government spending), the market clearing condition (19) implies that for output to be stable, consumption of both types of households must move in opposite direction. Since, according to our calibration, impatient households are relatively poor and hence consume less and work more, it is their consumption that goes up and that of patient households declines.

\textsuperscript{9}In Figure 1 we do not show the evolution of the LTV ratio as its path is not unique. This is because, from the collateral constraint (13), the LTV ratio depends not just on (uniquely determined) credit and the interest rate, but also on house prices. As we show in the proof of Proposition 1, there are many paths of house prices consistent with constant inflation and output.
To engineer the required move in household spending, the central bank raises the interest rate and hence depresses savers’ consumption while the macroprudential authority adjusts the LTV requirement allowing for an increase in credit and hence in borrowers’ expenditures. The market clearing condition (19) and restriction (21) imply that next period, when productivity is back to its long-term average, both agents’ consumption must return to their steady state levels. However, for this to happen, credit must keep increasing as loans taken in the previous period need to be repaid with interest, see the budget constraint of impatient households (12). Actually, every new period credit must increase further, which eventually leads to an explosion.

These findings suggest that defining monetary and macroprudential policy goals as strict stabilization of inflation and output gap is incompatible with what these two policies can jointly achieve. Interestingly, this conflict is not present if we assign the same two objectives to the monetary and fiscal authorities, the latter controlling government spending. We can state this result in the following conjecture.

**Conjecture 2.** In the benchmark macro-financial NK model, there always exist a stable equilibrium in which monetary and fiscal policy fully stabilize inflation and the output gap.

While no analytical proof for this statement can be offered, we show in the Appendix how it can be substantiated using a numerical optimization procedure. Hence, if the fiscal authority appropriately adjusts its expenditures, fluctuations in economic activity can be perfectly targeted without jeopardizing the price stability objective. Overall, from this part of our analysis fiscal policy emerges as a better complement to monetary policy if the macroeconomic goal is to simultaneously target inflation and the output gap.

### 3.2 Credit stabilization

We have seen that macroprudential policy does not help to resolve the classical inflation-output volatility tradeoff. However, this policy is usually thought of as designed to stabilize the financial sector rather than the real economy. We start by checking how changes in the LTV ratio can be combined with adjustments in the interest rates to simultaneously target inflation and credit. Our finding can be summarized in the following proposition.

**Proposition 3.** In the benchmark macro-financial NK model, a stable equilibrium in which monetary and macroprudential policy achieve full stabilization of inflation and the credit gap exists only if the role of credit in the economy is sufficiently small.

What the proposition tells us is that, under some model parametrization, perfect targeting of inflation and credit is possible without leading to explosive behavior. In the proof of this proposition documented in the Appendix, we show that this parametrization must be such
that
\[
\frac{\varphi \omega_c + \omega_n}{\varphi (1 - \omega_c - \omega_g) + 1 - \omega_n} \leq \frac{1 - \beta P}{2 \varphi} \left( 1 + \frac{\varphi + 1}{A_x} \right)
\] (22)

Holding other parameters fixed at our baseline calibration values, this restriction is satisfied for the share of impatient households \(\omega\) at most 0.077 or, alternatively, housing weight in utility \(A_x\) no more than 0.011. These threshold values imply (annualized) credit-to-GDP ratio of around 0.14 and 0.12, respectively, which is well below that observed in the data. Hence, perfect targeting of inflation and credit using monetary and macroprudential policy is possible only in a low credit environment.

As before, we illustrate these findings using the impulse response analysis, where we again assume the policy targets to be zero to facilitate the exposition. Figure 2 plots the reactions to a one-period positive technology shock for two alternative model parametrizations, differing in the values of \(\omega\) such that condition (22) is satisfied (low credit environment) or not (high credit environment). It can be seen that, if credit is low, the effects of the technology shock eventually die out. In the opposite case, the economy experiences oscillations whose amplitude increases over time.

The reason for this different behavior is as follows. First, recall that, following an increase in productivity, wages must go up to keep the price level stable, which requires an increase in an appropriately weighted sum of aggregate demand components, see equation (20). If credit is to be kept constant, this increase cannot be based on impatient households’ consumption only and hence monetary policy must be eased to make patient households consume more. However, a decrease in the interest rate implies an increase in impatient households’ disposable income next period as the cost of servicing their debt goes down. If credit is constant, this increase cannot be smoothed out and their consumption increases, which adds to wage and cost pressure. To prevent a rise in inflation, the central bank interest rates must go up, which means lower disposable income of borrowers next period, the need to relax the monetary policy, and so on and so forth.

If credit is low, the impact of fluctuations in the borrowing cost on impatient households’ income is limited, changes in their spending do not translate into big swings in aggregate demand and marginal cost, and hence the central bank does not need to make strong policy rate adjustments. If credit is high, these adjustments increase over time, which eventually leads to an explosion in the interest rate and some other macrovariables.

As previously, we compare these outcomes to those that can be achieved using fiscal rather than macroprudential policy as a complement to monetary policy. Our finding can be summarized by the following proposition.

**Proposition 4.** In the benchmark macro-financial NK model, there always exist a stable equilibrium in which monetary and fiscal policy fully stabilize inflation and the credit gap.

More precisely, as we prove in the Appendix, appropriate adjustments in government
spending and the short-term interest rate can ensure perfect stabilization of both inflation and credit at any exogenous targets without making the economy explode. Overall, one can conclude that while macroprudential policy has clear limits in ensuring stable credit without jeopardizing the standard price stability objective pursued by the central bank, the use of fiscal policy in this context is possible.

### 3.3 House price stabilization

Finally, we investigate the ability of macroprudential policy to target house prices if the central bank strictly adheres to the (consumer) price stability objective. We find by simulating the model for various parameter values that both outcomes, i.e. stable or unstable equilibria, are possible. Unfortunately, and unlike in the inflation and credit stabilization scenario, a tractable condition similar to (22) cannot be derived.\(^{10}\) Hence, we have to resort to numerical simulations.

Again, it turns out that the key parameters are those affecting the amount of credit in the economy. Figure 3 illustrates how the existence of a stable equilibrium depends on the relative size of impatient households \(\omega\) and weight of housing in utility \(A_\chi\). A non-explosive equilibrium exists only if either of the two parameters is sufficiently small, which means that the role of credit in the economy is not too large. This time, however, for parametrizations implying realistic ratios of credit and housing to GDP (our benchmark calibration), perfect targeting of inflation and house prices is possible.

Figure 4 plots how our model economy responds to a one-period productivity shocks for two different values of \(\omega\) chosen such that each of them falls into a distinct stability region, assuming that the short-term interest rate and LTV ratio are adjusted such that inflation and house prices are constant. If credit is low on average (low \(\omega\)), the economy eventually goes back to its steady state equilibrium. In a high credit environment (high \(\omega\)) there is no stable equilibrium. Also, and unlike in the case of unstable equilibria considered in Figures 1 and 2, there are infinitely many explosive paths for the key macrovariables so we plot just one of them that features the same initial response of impatient households’ consumption.

To understand why these paths are so different, it is instructive to first consider what kind of adjustments in other variables are needed to ensure constant house prices. To this end, let us rewrite the housing Euler equation (14) after substituting for \(\hat{\Theta}_t\) and \(\hat{m}_t\) using the impatient households Euler condition (11) and the collateral constraint (13), respectively

\[
\hat{p}_\chi,t = \beta_I \mathbb{E}_t \{ \hat{p}_{\chi,t+1} \} + (1 - \beta_P) \hat{c}_I,t - \beta_I (\hat{R}_t - \mathbb{E}_t \{ \hat{n}_{t+1} \}) + (\beta_P - \beta_I) \hat{l}_t
\]

Hence, house prices depend negatively on the real interest rate and positively on credit.

\(^{10}\)More precisely, the eigenvalues of the dynamic system implied by the model equations still can be derived analytically, but they are given by very involved formulas.
and impatient households’ consumption. Further, note that borrowers’ budget constraint (12) implies a positive relationship between their consumption and credit. This means that for house prices to remain unchanged, loans and the interest rate must move in the same direction.

Recall that according to the Phillips curve and formula (20), for inflation to remain unchanged, the appropriately weighted sum of both households’ consumption needs to increase. This can be achieved by a decrease in the short term interest rate, which boosts patient agents’ spending, and some tightening in credit to offset an upward pressure on house prices (see equation (23)), which leads to a drop in borrowers’ consumption. As in the case considered in the previous section, if credit is low on average, this monetary policy easing (and hence drop in credit cost) does not have a big impact on aggregate spending and wage pressure next period. In a high credit environment, the drop in interest cost paid by borrowers has a large impact on aggregate demand next period and, to prevent inflation, has to be counteracted by further tightening of credit conditions, which, in line with equation (23), requires further monetary policy easing to prevent house prices from falling. This self-propelling mechanism eventually leads to an explosion.

We finally check how these same objectives can be met with the monetary-fiscal policy mix. We can formulate our conclusions in the following proposition, which is proved in the Appendix.

**Proposition 5.** In the benchmark macro-financial NK model, there always exist a stable equilibrium in which monetary and fiscal policy fully stabilize inflation and the house price gap.

Hence, independent of the model parametrization, using the short-term interest rate and government spending to target inflation and house prices is possible as it does not lead to explosive behavior. Overall, one can conclude that both macroprudential and fiscal policy are efficient in complementing monetary policy when the primary stabilization objectives are defined as inflation and house price stability, but the monetary-macroprudential policy mix may fail in a large credit environment.

### 3.4 The case of non-rigid savers’ demand for housing

All results discussed in this section are based on the model variant in which patient households’ housing demand is assumed to be rigid. This assumption combined with fixed total housing stock effectively shuts down any trade in housing between the two types of agents, which renders the model more tractable and the propositions relatively easy to prove. In this subsection we briefly discuss how flexible housing demand changes our main findings regarding macroprudential policy. To this end, we assume that housing utility of patient
agents is of the same log form as the one for impatient households.\textsuperscript{11}

Starting with the inflation-output volatility tradeoff, we find our conclusions unaffected. It is not possible to use the short-term interest rate and the LTV ratio to fully stabilize inflation and the output gap such that the model economy does not explode. As regards the ability of macroprudential policy to target credit if the central bank fully stabilizes inflation, our findings are qualitatively similar to those derived for the benchmark model, i.e. there exist a stable equilibrium if the role of credit is sufficiently small. This time, however, the threshold leverage in the economy is much higher. In particular, our benchmark calibration is consistent with a stable equilibrium in which monetary and macroprudential policy meet their objectives defined as inflation and credit gap stability. Finally, allowing for flexible housing demand of lenders makes the conclusions regarding house price targeting stronger as this time an attempt to achieve it using the LTV ratio always leads to instability if the central bank keeps inflation constant.

Summing up, our general findings on the limited power of macroprudential policy in complementing monetary policy in achieving standard macroeconomic or additional financial stabilization objectives are robust to dropping the assumption on rigid housing demand of patient agents.

3.5 Other shocks and time-varying inflation target

We have already noted that the qualitative results discussed above do not depend on the type of aggregate uncertainty.\textsuperscript{12} This is because, as it becomes clear by looking at the construction of proofs presented in the Appendix, these results are derived from the properties of the dynamic systems where no distinction needs to be made between various types of shocks. For example, assume that our model economy is hit not only by productivity, but also by some cost-push (e.g. markup) shocks. Then the relevant target for output might be its flexible price level that would arise in the absence of cost-push shocks (as in Smets and Wouters, 2003), with the LTV ratio responding optimally according to some welfare criterion. This target output can be approximated by a function depending on current and past productivity shocks. Our findings in this context imply that monetary and macroprudential policy cannot simultaneously close such defined output gap and keep inflation at its target.

Similar considerations lead to a conclusion that our findings can be generalized to any non-zero and time-varying target for inflation, as long as it can be expressed as a function of current and past shocks hitting the economy. This might be a relevant case when the policy

\textsuperscript{11}Analytical derivations and simulations underlying the findings discussed below are available from the author upon request.

\textsuperscript{12}Naturally, we need to rule out shocks affecting the economy in a way that is observationally equivalent to changes in one of the policy instruments. An example is a shock to the value of collateral that shows up only on the right-hand side of equation (4), and whose effects can be completely eliminated by appropriate adjustments in the LTV ration $m_t$. 

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makers try to target more than two variables, in which case strict inflation stabilization might not not be optimal.

4 Conclusions

In this paper we have analyzed how macroprudential policy that uses the LTV ratio as its instrument can contribute to macroeconomic stability by helping the central bank in achieving a selection of alternative stabilization objectives. According to our results, what such defined policy can achieve has important limits. In particular, macroprudential policy does not help to resolve the inflation-output volatility tradeoff nor allows to achieve one additional financial target related to either credit or house price without conflicting with the price stability goal. In contrast, fiscal policy based on appropriate government spending adjustments can be combined with monetary policy to meet either of the three dual stabilization objectives.

While the transmission of these two alternative policies is inherently different, one feature seems to be key in understanding their different performance when combined with monetary policy. Similarly to the effects of monetary policy on savers, adjustments in the LTV ratio affect the ability of financially constrained borrowers to smooth their consumption over time, and hence this policy can be thought of as affecting mainly intertemporal allocations. In contrast, by directly affecting income of taxpayers, government spending has an intratemporal flavor and hence is a better complement to monetary policy.

Naturally, our results do not mean that LTV-based macroprudential policy cannot be useful, especially in preventing financial crises. However, one should not expect that implementing it at a business cycle frequency will resolve the key macroeconomic or macrofinancial stabilization tradeoffs.

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Tables and figures

Table 1: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_P$</td>
<td>0.995</td>
<td>Discount factor, patient HHs</td>
</tr>
<tr>
<td>$\beta_I$</td>
<td>0.99</td>
<td>Discount factor, impatient HHs</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.25</td>
<td>Share of impatient HHs in population</td>
</tr>
<tr>
<td>$A_\chi$</td>
<td>0.045</td>
<td>Weight on housing in utility</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>2</td>
<td>Inverse of Frisch elasticity of labor supply</td>
</tr>
<tr>
<td>$\mu$</td>
<td>1.2</td>
<td>Steady state product markup</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.75</td>
<td>Calvo probability for prices</td>
</tr>
<tr>
<td>$g_y$</td>
<td>0.165</td>
<td>Share of government spending in output</td>
</tr>
</tbody>
</table>

Figure 1: Inflation and output stabilization with monetary and macroprudential policy

Note: The plots present the responses to a one-period positive productivity shock, assuming that the central bank and macroprudential authority fully stabilize inflation and output.
Figure 2: Inflation and credit stabilization with monetary and macroprudential policy in low and high credit environment

Note: The plots present the responses to a one-period positive productivity shock, assuming that the central bank and macroprudential authority fully stabilize inflation and credit.

Figure 3: Equilibrium stability regions if monetary and macroprudential policy fully stabilize inflation and house prices

Note: The plot present the equilibrium stability regions as a function of two key parameters, both presented on the log scale. The gray spot indicates our baseline calibration.
Figure 4: Inflation and house price stabilization with monetary and macroprudential policy in low and high credit environment

Note: The plots present the responses to a one-period positive productivity shock, assuming that the central bank and macroprudential authority fully stabilize inflation and house prices. To increase the figure’s legibility, the response of patient households’ consumption in a low credit environment is multiplied by a factor of ten.
Appendix

A.1 Model equations

In this section of the Appendix we present a full list of equations making up the benchmark NK macrofinancial model. The variables without time subscripts denote their steady state values.

Households

Euler equation for patient households

\[
c_{P,t}^{-1} = \beta_P \mathbb{E}_t \left\{ c_{P,t+1}^{-1} \pi_t^{-1} \right\} R_t
\]  

(A.1)

Impatient households’ budget constraint

\[
c_{I,t} + p_{\chi,t}(\chi_{I,t} - \chi_{I,t-1}) + R_{t-1}l_{t-1}\pi_t^{-1} = l_t + w_t n_{I,t}
\]  

(A.2)

Collateral constraints

\[
R_t l_t = m_t p_{\chi,t} \chi_{I,t}
\]  

(A.3)

Euler equations for impatient households

\[
c_{I,t}^{-1} = R_t \left( \beta_I \mathbb{E}_t \left\{ c_{I,t+1}^{-1} \pi_{t+1}^{-1} \right\} + \Theta_t \right)
\]  

(A.4)

Rigid housing demand of patient households

\[
\chi_{P,t} = \chi_P
\]  

(A.5)

Housing Euler equation for impatient households

\[
c_{I,t}^{-1} P_{\chi,t} = A_\chi \chi_I^{-\sigma} + \beta_I \mathbb{E}_t \left\{ c_{I,t+1}^{-1} P_{\chi,t+1} \right\} + \Theta_t m_t P_{\chi,t}
\]  

(A.6)

Labor supply (for \( i = \{I, P\} \))

\[
w_i c_{i,t}^{-1} = A_n n_{i,t}^{\nu}
\]  

(A.7)

Labor aggregate

\[
n_t = \omega n_{I,t} + (1 - \omega) n_{P,t}
\]  

(A.8)
Firms

Marginal cost

\[ mc_t = \frac{w_t}{\varepsilon_t} \]  \hspace{1cm} (A.9)

Optimal price set by reoptimizing firms

\[ \tilde{p}_t = \mu \frac{\Omega_t}{\Upsilon_t} \]  \hspace{1cm} (A.10)

Auxiliary functions for optimal price

\[ \Omega_t = c_{P_t}^{-1} mc_t y_t + \beta P_t \theta E_t \left\{ \pi_{t+1}^{\mu+1} \Omega_{t+1} \right\} \]  \hspace{1cm} (A.11)

\[ \Upsilon_t = c_{P_t}^{-1} y_t + \beta P_t \theta E_t \left\{ \pi_{t+1}^{\mu+1} \Upsilon_{t+1} \right\} \]  \hspace{1cm} (A.12)

Price indexes

\[ 1 = \theta \left( \frac{\pi_t}{\pi_{t}} \right) \frac{1}{1-\mu} + (1 - \theta) \frac{1}{\tilde{p}_t^{1-\mu}} \]  \hspace{1cm} (A.13)

Market clearing

Goods market

\[ y_t = \omega c_{I,t} + (1 - \omega) e_{P,t} + g_t \]  \hspace{1cm} (A.14)

Aggregate output

\[ y_t \Delta_t = \varepsilon_t m_t \]  \hspace{1cm} (A.15)

Price dispersion index

\[ \Delta_t = \theta \Delta_{t-1} \pi_t^{\mu+1} + (1 - \theta) \frac{1}{\tilde{p}_{H,t}^{\mu+1}} \]  \hspace{1cm} (A.16)

Housing market

\[ \chi = \omega_I \chi_{I,t} + (1 - \omega_I) \chi_{P,t} \]  \hspace{1cm} (A.17)

A.2 Inflation and output gap stabilization using monetary and macroprudential policy (proof of Proposition 1)

We prove Proposition 1 by showing that stabilization of inflation and output at some exogenous targets using the short-term interest rate and the LTV ratio is inconsistent with the existence of a stable rational expectations equilibrium.

Let us denote any linear function of current and past exogenous variables up to time \( t \) as \( exo_t = A(L)\hat{\varepsilon}_t \) where \( A(L) = a_0 + a_1 L + a_2 L^2 + ... \) is any lag polynomial
in $L$ with $Lx_t = x_{t-1}$ for any variable $x$. First note that if output is at its target at all times, the market clearing condition (19) implies

$$\omega_c \hat{c}_{I,t} + (1 - \omega_c - \omega_g) \hat{c}_{P,t} + \omega_g \hat{y}_t = exo_t$$  \hspace{1cm} (A.18)

If additionally inflation is at the target at all times, the Phillips curve implies $\hat{w}_t = \hat{\epsilon}_t$ and hence equation (20) can be rewritten as

$$\omega_n \hat{c}_{I,t} + (1 - \omega_n) \hat{c}_{P,t} = exo_t$$  \hspace{1cm} (A.19)

With inactive fiscal policy ($\hat{g}_t = 0$), these two equations which can be solved for $\hat{c}_{I,t}$ and $\hat{c}_{P,t}$ as functions of exogenous shocks only.

Further, the Euler equation for patient households (10) implies that $\hat{R}_t$ can be written as a function of exogenous shocks only, which together with the Euler equation for impatient households (11) leads to the same result for $\hat{\Theta}_t$. Then, after using the collateral constraint (13) to eliminate $\hat{m}_t$, our benchmark NK macrofinancial model can be reduced to the following three equations

$$\hat{R}_t = exo_t$$  \hspace{1cm} (A.20)

$$\frac{1}{\beta_p} (\hat{R}_{t-1} + \hat{l}_{t-1}) - \hat{l}_t = exo_t$$  \hspace{1cm} (A.21)

$$\hat{p}_{\chi,t} - \beta_I \mathbb{E}_t \{ \hat{p}_{\chi,t+1} \} + (\beta_I - \beta_P) \hat{l}_t = exo_t$$  \hspace{1cm} (A.22)

It is clear by combining equations (A.20) and (A.21) that credit is explosive in response to shocks. More formally, system (A.20)-(A.22) can be cast in matrix notation

$$\Gamma_0 \begin{bmatrix} \hat{R}_t \\ \hat{l}_t \\ \mathbb{E}_t \hat{p}_{\chi,t+1} \end{bmatrix} = \Gamma_1 \begin{bmatrix} \hat{R}_{t-1} \\ \hat{l}_{t-1} \\ \hat{p}_{\chi,t} \end{bmatrix} + Exo_t$$  \hspace{1cm} (A.23)

where

$$\Gamma_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \beta_P - \beta_I & \beta_I \end{bmatrix}$$ \hspace{1cm} $$\Gamma_1 = \begin{bmatrix} 0 & 0 & 0 \\ \frac{1}{\beta_P} & \frac{1}{\beta_P} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and $Exo_t$ denotes any vector of functions $exo_t$ of appropriate dimension.

The determinant of $\Gamma_0$ is equal to $\beta_I > 0$, which allows us to write

$$\begin{bmatrix} \hat{R}_t \\ \hat{l}_t \\ \mathbb{E}_t \hat{p}_{\chi,t+1} \end{bmatrix} = \Gamma \begin{bmatrix} \hat{R}_{t-1} \\ \hat{l}_{t-1} \\ \hat{p}_{\chi,t} \end{bmatrix} + Exo_t$$  \hspace{1cm} (A.24)
The eigenvalues of $\Gamma \equiv \Gamma_0^{-1} \Gamma_1$ are $[ \beta_t^{-1} \beta_P^{-1} 0 ]$. Hence, there are two eigenvalues outside the unit circle and only one forward-looking variable in the associated system (A.23), which means that no stable equilibrium exists, see Blanchard and Kahn (1980).

Note that while the paths of $\hat{R}_t$ and $\hat{l}_t$ are uniquely determined, there are many paths of $\hat{p}_{\chi,t}$ consistent with system (A.20)-(A.22), all of which are explosive.

A.3 Inflation and output gap stabilization using monetary and government spending policy (substantiation of Conjecture 2)

In this section we show that perfect stabilization of both inflation and output at some exogenous targets using the short-term interest rate and government spending is consistent with the existence of a stable rational expectations equilibrium. As in the proof of Proposition 1, first note that if output is at its target at all times, the market clearing condition (19) implies

$$\omega_c \hat{c}_{I,t} + (1 - \omega_c - \omega_g) \hat{c}_{P,t} + \omega_g \hat{g}_t = \text{exo}_t$$

where $\text{exo}_t$ denotes any linear function of exogenous variables up to time $t$. If additionally inflation is at the target at all times, equation (20) can be written as

$$\omega_n \hat{c}_{I,t} + (1 - \omega_n) \hat{c}_{P,t} = \text{exo}_t$$

which allows us to solve for $\hat{c}_{P,t}$ as functions of $\hat{c}_{I,t}$ and exogenous variables only:

$$\hat{c}_{P,t} = -b \hat{c}_{I,t} + \text{exo}_t$$

where $b \equiv \frac{1 - \omega_n}{\omega_n} > 0$.

Plugging this condition into the model and taking into account that macroprudential policy is passive ($\hat{m}_t = 0$) gives the following system of equations

$$b \hat{c}_{I,t} = b \mathbb{E}_t \{ \hat{c}_{I,t+1} \} + \hat{R}_t + \text{exo}_t$$

$$\beta_P \hat{c}_{I,t} = \beta_I \mathbb{E}_t \{ \hat{c}_{I,t+1} \} - \beta_P \hat{R}_t - (\beta_P - \beta_I) \hat{\Theta}_t$$

$$s_c \hat{c}_{I,t} + \frac{1}{\beta_P} (\hat{R}_{t-1} + \hat{l}_{t-1}) - \hat{l}_t = \text{exo}_t$$

$$\hat{R}_t + \hat{l}_t = \hat{p}_{\chi,t}$$

$$(1 - \beta_P + \beta_I) \hat{p}_{\chi,t} = \beta_I \mathbb{E}_t \{ \hat{p}_{\chi,t+1} \} + \hat{c}_{I,t} - \beta_I \mathbb{E}_t \{ \hat{c}_{I,t+1} \} + (\beta_P - \beta_I) (\hat{\Theta}_t + \hat{m}_t)$$

where $s_c = \frac{c_t}{l_t} + \frac{s_{nl}}{\sigma_n}$.

After using the and borrowers’ Euler equation (A.4) to eliminate $\hat{\Theta}_t$, the collateral constraint (13) to eliminate $\hat{l}_t$, and the saver’s Euler equation to eliminate $\hat{R}_t$, our benchmark
NK macrofinancial model can be reduced to the following two equations

\[(b + s_c)\hat{c}_{I,t} + \frac{1}{\beta_P} \hat{p}_{x,t-1} - \hat{p}_{x,t} - b\mathbb{E}_t \{\hat{c}_{I,t+1}\} = \text{exo}_t \]  
(A.33)

\[(\beta_P(1 + b) - 1)\hat{c}_{I,t} + (1 - \beta_P + \beta_I)\hat{p}_{x,t} - \beta_I \mathbb{E}_t \{\hat{p}_{x,t+1}\} - b\beta_P \mathbb{E}_t \{\hat{c}_{I,t+1}\} = \text{exo}_t \]  
(A.34)

Defining \(\hat{p}_{x,t} \equiv \hat{p}_{x,t-1}\) allows us to cast the system (A.33)-(A.34) in matrix notation

\[
\begin{bmatrix}
\hat{p}_{x,t+1} \\
\mathbb{E}_t \hat{c}_{I,t+1} \\
\mathbb{E}_t \hat{p}_{x,t+1}
\end{bmatrix} = \Gamma_1 \begin{bmatrix}
\hat{p}_{x,t} \\
\hat{c}_{I,t} \\
\hat{p}_{x,t}
\end{bmatrix} + \text{exo}_t 
\]  
(A.35)

where

\[
\Gamma_0 = \begin{bmatrix}
1 & 0 & 0 \\
0 & b & 0 \\
0 & b\beta_P & \beta_I
\end{bmatrix} \quad \Gamma_1 = \begin{bmatrix}
0 & 0 & 1 \\
\frac{1}{\beta_P} & b + s_c & -1 \\
0 & \beta_P(1 + b) & 1 - \beta_P + \beta_I
\end{bmatrix}
\]

The determinant of \(\Gamma_0\) is equal to \(b\beta_I > 0\), which allows us to write

\[
\begin{bmatrix}
\hat{p}_{x,t+1} \\
\mathbb{E}_t \hat{c}_{I,t+1} \\
\mathbb{E}_t \hat{p}_{x,t+1}
\end{bmatrix} = \Gamma \begin{bmatrix}
\hat{p}_{x,t} \\
\hat{c}_{I,t} \\
\hat{p}_{x,t}
\end{bmatrix} + \text{exo}_t 
\]  
(A.36)

The three eigenvalues of \(\Gamma \equiv \Gamma_0^{-1}\Gamma_1\) can be derived analytically but the solution formulas are complicated and hence Conjecture 2 cannot be proved formally. However, one can check numerically that at least one of these eigenvalues lies within the unit circle, for any admissible model parameter values. Hence, since there are two forward-looking variables in the associated system (A.35), a stable equilibrium exists, see Blanchard and Kahn (1980).

**A.4 Inflation and credit gap stabilization using monetary and macroprudential policy (proof of Proposition 3)**

We prove Proposition 3 by showing that perfect stabilization of both inflation and credit at some exogenous targets using the short-term interest rate and LTV ratio is consistent with the existence of a stable rational expectations equilibrium only if the model parameters satisfy a certain restriction implying relatively minor role of credit in the economy.

If inflation is at the target at all times, equation (20) and the Phillips curve imply

\[
\varphi(\omega_c \hat{c}_{I,t} + (1 - \omega_c - \omega_g)\hat{c}_{P,t} + \omega_g \hat{g}_t) + \omega_n \hat{c}_{I,t} + (1 - \omega_n)\hat{c}_{P,t} = \text{exo}_t 
\]  
(A.37)
which allows us to express patient households’ consumption as function of that of impatient ones and exogenous variables

\[ \dot{c}_{p,t} = -a \dot{c}_{I,t} + exo_t \]  

(A.38)

where \( a \equiv \frac{\omega_0 + \omega_n}{\varphi(1-\omega_c-\omega_p)+1-\omega_n} > 0 \) and \( exo_t \) denotes any linear function of exogenous variables up to time \( t \).

Plugging this condition into the model equations and restricting credit to be at its exogenous target at all times leads to the following system of equations

\[-a \dot{c}_{I,t} + a E_t \{ \dot{c}_{I,t+1} \} + \dot{R}_t = exo_t \]  

(A.39)

\[ sc \dot{c}_{I,t} + \frac{1}{\beta_P} \dot{R}_{t-1} = exo_t \]  

(A.40)

\[ \dot{c}_{I,t} - \dot{p}_{\chi,t} + \beta_I E_t \{ -\dot{c}_{I,t+1} + \dot{p}_{\chi,t+1} \} + (\beta_P - \beta_I) (\dot{\Theta}_t + \dot{m}_t + \dot{p}_{\chi,t}) = exo_t \]  

(A.41)

\[ \beta_P (\dot{c}_{I,t} + \dot{R}_t) - \beta_I E_t \{ \dot{c}_{I,t+1} \} + (\beta_P - \beta_I) \dot{\Theta}_t = exo_t \]  

(A.42)

\[ \dot{R}_t = \dot{m}_t + \dot{p}_{\chi,t} + exo_t \]  

(A.43)

where \( sc \equiv \frac{c_t}{\varphi} + \frac{wn_t}{\varphi t} > 0 \).

If we use equation (A.40) to eliminate \( \dot{c}_{I,t} \), equation (A.42) to eliminate \( \dot{\Theta}_t \), and equation (A.43) to eliminate \( \dot{m}_t \), the system reduces to two equations in two endogenous variables

\[ \frac{a}{sc \beta_P} \dot{R}_{t-1} + (1 - \frac{a}{sc \beta_P}) \dot{R}_t = exo_t \]  

(A.44)

\[ -\frac{1 - \beta_P}{sc \beta_P} \dot{R}_{t-1} - \dot{p}_{\chi,t} - \beta_I \dot{R}_t + \beta_I E_t \{ \dot{p}_{\chi,t+1} \} = exo_t \]  

(A.45)

Casting it in matrix notation yields

\[ \Gamma_0 \begin{bmatrix} R_t \\ E_t \dot{p}_{\chi,t+1} \end{bmatrix} = \Gamma_1 \begin{bmatrix} R_{t-1} \\ \dot{p}_{\chi,t} \end{bmatrix} + Exo_t \]  

(A.46)

where

\[ \Gamma_0 = \begin{bmatrix} 1 - \frac{a}{sc \beta_P} & 0 \\ -\beta_I & \beta_I \end{bmatrix} \quad \Gamma_1 = \begin{bmatrix} -\frac{a}{sc \beta_P} & 0 \\ \frac{1 - \beta_P}{sc \beta_P} & 1 \end{bmatrix} \]

Note that the determinant of \( \Gamma_0 \) is \((1 - \frac{a}{sc \beta_P})\beta_I\), which is non-zero except for very special parametrization, so we can write

\[ \begin{bmatrix} R_t \\ E_t \dot{p}_{\chi,t+1} \end{bmatrix} = \Gamma \begin{bmatrix} R_{t-1} \\ \dot{p}_{\chi,t} \end{bmatrix} + Exo_t \]  

(A.47)

The two eigenvalues of \( \Gamma \) are \( 1/\beta_I \) and \( a/(a - \beta_P sc) \). The first one clearly lies outside the
unit circle. Since there is one forward-looking variable in system (A.46), and both $a$ and $s_c$ are strictly positive, for a stable equilibrium to exist we must have $2a \leq \beta_p s_c$.

Let us now work more on the formula for $s_c$. First note that the Euler equation (A.1) evaluated in the steady state implies

$$R = \beta_p^{-1}$$

(A.48)

This together with the Euler equation for impatient agents (A.4) yields

$$\Theta = c_I^{-1}(\beta_p - \beta_I)$$

(A.49)

This formula can be used to substitute for $\Theta$ in the housing Euler equation (A.6), which gives

$$1 - \beta_p = \frac{A_x c_I}{p \chi_I}$$

(A.50)

After using the collateral constraint (A.3) to substitute for $p \chi_I$ we obtain

$$\frac{c_I}{I} = \frac{1 - \beta_p}{A_x \beta_p}$$

(A.51)

Next note that the budget constraint of impatient households (A.2) evaluated at the steady state implies

$$\frac{wn}{I} = \frac{c_I}{I} + \beta_p^{-1} - 1$$

(A.52)

Now we are ready to derive the formula for $s_c$ as a function of deep model parameters

$$s_c = \frac{c_I}{I} + \varphi^{-1} \frac{wn}{I} = (1 + \varphi^{-1}) \frac{c_I}{I} + \varphi^{-1}(\beta_p^{-1} - 1) = \frac{1 - \beta_p}{\varphi \beta_p} \left(1 + \frac{\varphi + 1}{A_x}\right)$$

(A.53)

Plugging this equation into condition $2a \leq \beta_p s_c$ gives

$$\frac{\varphi \omega_c + \omega_n}{\varphi(1 - \omega_c - \omega_n) + 1 - \omega_n} \leq \frac{1 - \beta_p}{2 \varphi} \left(1 + \frac{\varphi + 1}{A_x}\right)$$

which is restriction (22) in the main text.

### A.5 Inflation and credit gap stabilization using monetary and government spending policy (proof of Proposition 4)

Proposition 4 is proved by demonstrating that perfect stabilization of both inflation and credit at some exogenous targets using the short-term interest rate and government spending is consistent with existence of a stable equilibrium.

First note that, if inflation and credit are both at their targets at all times and macro-prudential policy is inactive (i.e. not responding to endogenous variables), the model can be
reduced to the following system of equations

\[
\hat{c}_{P,t} - \mathbb{E}_t \{ \hat{c}_{P,t+1} \} + \hat{R}_t = \text{exo}_t \quad (A.54)
\]

\[
s_c \hat{c}_{I,t} + \frac{1}{\beta_P} \hat{R}_{t-1} = \text{exo}_t \quad (A.55)
\]

\[
\hat{c}_{I,t} - \hat{p}_{\chi,t} + \beta_I \mathbb{E}_t \{-\hat{c}_{I,t+1} + \hat{p}_{\chi,t+1}\} + (\beta_P - \beta_I)(\hat{\Theta}_t + \hat{p}_{\chi,t}) = \text{exo}_t \quad (A.56)
\]

\[
\beta_P(\hat{c}_{I,t} + \hat{R}_t) - \beta_I \mathbb{E}_t \{\hat{c}_{I,t+1}\} + (\beta_P - \beta_I)\hat{\Theta}_t = \text{exo}_t \quad (A.57)
\]

\[
\hat{R}_t = \hat{p}_{\chi,t} + \text{exo}_t \quad (A.58)
\]

where \( s_c \equiv \frac{c_I}{t} + \frac{\ln I}{C_I} > 0 \) and \( \text{exo}_t \) denotes any linear function of exogenous variables showing up in the model up to time \( t \).

Eliminating \( \hat{\Theta}_t, \hat{R}_t \) and \( \hat{c}_{I,t} \) allows us to write

\[
\hat{c}_{P,t} - \mathbb{E}_t \{ \hat{c}_{P,t+1} \} + \hat{p}_{\chi,t} = \text{exo}_t \quad (A.59)
\]

\[
\frac{1 - \beta_P}{s_c \beta_P} \hat{p}_{\chi,t-1} + (1 + \beta_I)\hat{p}_{\chi,t} - \beta_I \mathbb{E}_t \{\hat{p}_{\chi,t+1}\} = \text{exo}_t \quad (A.60)
\]

After defining \( \hat{p}_{\chi,t} \equiv \hat{p}_{\chi,t-1} \), the system can be cast in matrix notation as follows

\[
\begin{bmatrix}
\hat{p}_{\chi,t+1} \\
E_t \hat{c}_{P,t+1} \\
E_t \hat{p}_{\chi,t+1}
\end{bmatrix}
= 
\begin{bmatrix}
\hat{p}_{\chi,t} \\
\hat{c}_{P,t} \\
\hat{p}_{\chi,t}
\end{bmatrix}
+ \text{exo}_t
\]

where

\[
\Gamma_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \beta_I \end{bmatrix} \quad \Gamma_1 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ \frac{1 - \beta_P}{s_c \beta_P} & 0 & 1 + \beta_I \end{bmatrix}
\]

Note that the determinant of \( \Gamma_0 \) is \( \beta_I > 0 \), hence we can write

\[
\begin{bmatrix}
\hat{p}_{\chi,t+1} \\
E_t \hat{c}_{P,t+1} \\
E_t \hat{p}_{\chi,t+1}
\end{bmatrix}
= \Gamma \begin{bmatrix}
\hat{p}_{\chi,t} \\
\hat{c}_{P,t} \\
\hat{p}_{\chi,t}
\end{bmatrix}
+ \text{exo}_t
\]

It is easy to verify that one of the eigenvalues of \( \Gamma = \Gamma_0^{-1} \Gamma_1 \) is 1, which is inside the unit circle, while the other two are given by the following formula

\[
\beta_I + 1 \pm \sqrt{(\beta_I + 1)^2 + \frac{4\beta_I(1-\beta_P)}{\beta_P s_c}} \quad \frac{1}{2\beta_I}
\]
Since there are two forward-looking variables in the reduced system (A.61), there exist at least one stable equilibrium, see Blanchard and Kahn (1980).

### A.6 Inflation and house price gap stabilization using monetary and government spending policy (proof of Proposition 5)

As in the case of Proposition 4, Proposition 5 is proved by demonstrating that perfect stabilization of both inflation and house prices at some exogenous targets using the short-term interest rate and government spending is consistent with existence of a stable equilibrium.

First use the assumption on zero deviation of inflation and house prices from their respective targets together with inactive macroprudential policy to reduce the model to the following system of equations

\[
\hat{c}_{P,t} - E_t \{\hat{c}_{P,t+1}\} + \hat{R}_t = \text{exo}_t \tag{A.63}
\]

\[
s_c \hat{c}_{I,t} + \frac{1}{\beta_P} (\hat{R}_{t-1} + \hat{l}_{t-1}) - \hat{l}_t = \text{exo}_t \tag{A.64}
\]

\[
\hat{c}_{I,t} - \beta_I E_t \{\hat{c}_{I,t+1}\} + (\beta_P - \beta_I) \hat{\Theta}_t = \text{exo}_t \tag{A.65}
\]

\[
\beta_P (\hat{c}_{I,t} + \hat{R}_t) - \beta_I E_t \{\hat{c}_{I,t+1}\} + (\beta_P - \beta_I) \hat{\Theta}_t = \text{exo}_t \tag{A.66}
\]

\[
\hat{R}_t + \hat{l}_t = \text{exo}_t \tag{A.67}
\]

where, as previously, \( s_c \equiv \frac{c_t}{l_t} + \frac{w_{It}}{c_t l_t} > 0 \) and \( \text{exo}_t \) denotes any linear function of exogenous variables up to time \( t \).

Now eliminate \( \hat{\Theta}_t \) and \( \hat{l}_t \) to obtain

\[
\hat{c}_{P,t} - E_t \{\hat{c}_{P,t+1}\} + \hat{R}_t = \text{exo}_t \tag{A.68}
\]

\[
s_c \hat{c}_{I,t} + \hat{R}_t = \text{exo}_t \tag{A.69}
\]

\[
(1 - \beta_P) \hat{c}_{I,t} - \beta_P \hat{R}_t = \text{exo}_t \tag{A.70}
\]

Since equations (A.69) and (A.70) can be solved for \( \hat{c}_{I,t} \) and \( \hat{R}_t \) as functions of exogenous variables only, we can write equation (A.68) as

\[
\hat{c}_{P,t} - E_t \{\hat{c}_{P,t+1}\} = \text{exo}_t \tag{A.71}
\]

There is a unit eigenvalue associated with this equation and it includes one forward-looking variable, hence there exist a stable equilibrium, see Blanchard and Kahn (1980).