Social Network Dynamics: Individual-Level Mechanisms and Aggregate Outcomes

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Abstract. Based on a calibrated computational multi-agent model with an overlapping generations structure, we investigate society-level consequences of creation and destruction of social ties at the individual level. The steady state of the simulated social network exhibits realistic small-world topology. We also find that societies whose social networks are relatively frequently reconfigured, display relatively higher social trust, willingness to cooperate, and economic performance – at the cost of lower social utility. Similar outcomes are found for societies where social tie dissolution is relatively weakly linked to family closeness.

Keywords: social network structure, social network dynamics, social trust, willingness to cooperate, economic performance, computational multi-agent model.

1. Introduction

The objective of this paper is to identify the key mechanisms explaining how social network structure affects individuals' social trust, willingness to cooperate, social utility and economic performance. To this end, we study how social networks give rise to the accumulation of social capital, defined as the aggregate of resources accessible to individuals through their social networks (Bourdieu, 1986), and how in turn social capital enables the creation of trust and cooperation. In consequence those elements govern how agents decide to create and sever their social ties. This
results in a dynamically changing social network. We analyze the properties of such a dynamic network and find that in the long run this evolution process leads to emergent small-world topology of social contacts, which is in line with topologies observed in real life.

Social network structure can have a sizable impact on individuals' social trust and willingness to cooperate as well as – ultimately – social utility and economic performance. Effects have been observed both at the individual level (e.g., individuals with more bridging social capital tend to be more willing to cooperate and economically better off), and at the aggregate level (e.g., societies that either are better connected or exhibit a lower frequency of local cliques, tend to record relatively better aggregate economic performance), see e.g. Coleman (1988), Dasgupta (1988), Putnam (2000), Inglehart and Baker (2000), Zak and Knack (2001), Burt (2005), Granovetter (2005). The studies cited above base their findings on a snapshot of network structure at one point in time. Social network structure is, however, far from static.

In this paper we propose a new agent-based model with a realistic demographic and social network structure, where the network evolves as individuals build new ties whereas some old ties are dissolved. We discuss the aggregate outcomes as well as individual-level correlations. The details of the model setup draw from our empirical findings for the Polish society (Growiec et al., 2017), based on a unique, detailed survey dataset. The evolving social network in steady-state of the simulation exhibits small-world topology – a type of network that is observed in real life. This emergent behavior of the system, to our knowledge, is a unique contribution of our model to the literature, at least among networks endogenously generated in multi-agent models with explicit economic and social motives guiding the creation and destruction of social ties. In particular, in our earlier work (Growiec et al., 2018) we have studied similar research questions under a static (fixed) network. In this paper we demonstrate how results from that paper generalize to the case of endogenous network creation. Additionally, considering an evolving system, apart from having more realistic assumptions, allows us to study social network dynamics, which is not possible for a static model.
We find that societies with a higher frequency of social tie creation and destruction, both per annum and within each individual’s lifetime, are – \textit{ceteris paribus} – more trustful and cooperative, and exhibit better economic performance. On the other hand, they display lower levels of bonding social capital and lower average social utility. Opposite effects are observed for societies where the durability of social ties is relatively strongly linked to agents' family closeness: they are less trustful and their economic performance is worse, but they imply more bonding social capital and social utility. We also find that societies where young individuals enter their adult life in relatively inclusive, non-clustered networks tend to be more trustful and exhibit better economic performance.

Hence, these results underscore that social network dynamics can be conducive to a trade-off between social utility and economic performance. In this paper we demonstrate that the trade-off can arise endogenously, driven by a few key characteristics of social network dynamics. This also means that the trade-off remains in force also when one accounts for finite lifespans of individuals in an overlapping generations setup and the possibility of social tie decay.

The remainder of the paper is structured as follows. In Sections 2 and 3 we motivate and lay out the model. In Section 4 we present the results and discuss them against the relevant literature. Section 5 concludes.

2. Conceptual framework of the model

2.1. Defining social capital

In this text we adopt Bourdieu’s (1986) definition of social capital: "\textit{Social capital is the aggregate of the actual or potential resources which are linked to possession of a durable network of more or less institutionalized relationships of mutual acquaintance and recognition – or in other words, to membership in a group – which provides each of its members with the backing of the collectivity-owned capital, a 'credential' which entitles them to credit, in the various senses of the word.}" (p. 128). This definition is widely shared in sociology (Lin, 2001; Kadushin, 2002; Li et al. 2005; Burt, 2005) and naturally allows to separate people's observable behavior (related to maintaining
social contacts) from latent traits such as trust or willingness to cooperate, that are considered as 
social capital correlates or outcomes. Moreover, under this definition we are able to link individuals’ 
economic performance and social utility to the structure of their social network by the fact that they 
can access resources via their social contacts (Bourdieu, 1986; Lin, 2001).

In this paper we operationalize Bourdieu's (1986) approach by considering four key 
dimensions of individuals' social capital: (i) degree, (ii) centrality, (iii) bridging and (iv) bonding social 
capital. To capture all four network characteristics independently, a minimal model has to explicitly 
acknowledge individuals' heterogeneity not only in terms of their position in the social network, but 
also in terms of additional individual traits. In order to do so we consider the following traits:

1) *family location* $f_i$, with the presumption that social ties between individuals who are close to 
each other in terms of $f_i$ represent (relatively strong and exclusive but economically less 
valuable) kinship ties whose aggregation represents the individual's stock of bonding social 
capital;

2) *agent type* $v_i$, with the presumption that social ties between individuals who are distant in 
terms of $v_i$ represent (relatively weak but economically profitable) bridging ties whose 
aggregation represents the individual's stock of bridging social capital. In Section 3 we discuss in 
detail the background literature motivating the assumption that contacts between dissimilar 
agents have higher economic potential.

3) *agent age* $a_i$ as empirical data (Growiec et al., 2017) shows that individuals activity in creating 
and breaking dissolving social ties varies during their life span.

We motivate our choice of the four social capital dimensions (degree, centrality, bridging 
and bonding social capital) as follows.

Inclusion of *degree* (the number of social ties of an individual) as a dimension of social 
capital is natural: *ceteris paribus* more network resources should be available to individuals who 
maintain more social ties.
Considering centrality as our second social capital dimension follows Burt's (1992) the "structural holes" argument. It states that, ceteris paribus, more resources are available to the individuals who are bridges between otherwise disconnected sub-networks as because they are often a part of a chain enabling the flow of resources in the network. This gives them a chance to exploit this position to gain extraordinary benefits.

Finally, not all network connections are equally valuable. Access to network resources depends on the fact to whom a person is connected. In the literature this is captured by introducing a differentiation between bridging social capital (social ties with dissimilar others) and bonding social capital (social ties with similar others), see e.g. Putnam (2000). According to the psychodynamic explanation, bonding social capital is formed to satisfy the safety need and support the status quo, whereas bridging social capital is formed to satisfy the efficacy need and enhance innovation (Bowlby, 1969; Greenberg, 1991; Kadushin, 2002, 2012). Because of this difference we view bridging and bonding social capital as distinct concepts and not just opposite sides of the same spectrum. Bridging social capital should be influencing individuals’ economic performance, while bonding social capital should translate to their social utility. It is also a well documented fact that bridging ties weaken or disappear much faster than bonding ones due to lack of embeddedness in social structure (Burt, 2000, 2002).

2.2. Channels of influence of social capital on economic performance and social utility

Granovetter (2005) observes that social networks are an effective source of feedback on individual behavior via the flow of information. Thus the level of an individual's trust and willingness to cooperate may depend on her position in the network. In consequence, social network structure may influence who cooperates with whom (affecting the decisions on whether we want to engage in a contact with a given stranger) and the individuals’ approach to economic exchanges (e.g. the level of willingness to exploit the other side of the contact), see e.g. Misztal (1996) and Field (2010). Those are natural channels influencing how social network structure translates to the outcomes of social interaction. For instance, as observed by Lazarsfeld and Merton (1954) and Lin (2001), dense sub-
networks are typically formed by similar agents, and lead to reputation formation and social control serving as substitutes of trust (Dasgupta, 1988). Conversely, in sparse networks individuals need more social trust to behave cooperatively as such networks are less effective in transmitting reputation information. On the other hand in dense networks social ties are often more redundant than in sparse networks in terms of access to resources (Granovetter, 2005). Similar considerations, via the transaction cost argument, translate to individual’s cooperativeness and thrift (Inglehart and Baker, 2000; Florida, 2004; Klapwijk and van Lange, 2009).

Following these observations, in our model social trust determines the probability that two individuals engage in economic interaction. Then the outcome of the contact is influenced by their willingness to cooperate. We model this relation using “prisoner's dilemma” game: both agents are better off when both cooperate than when both defect, but each of them is also individually tempted to defect. The model is calibrated so that an interaction where both agents defect is better than no interaction at all, but it is better not to interact at all than to interact, cooperate, and be cheated.

The above assumptions are justified by empirical evidence regarding the existence of causal links between social trust, cooperation and economic performance (see e.g. Knack and Keefer, 1997; Zak and Knack, 2001; Algan and Cahuc, 2010).

The relation between social capital, and social utility and economic performance, the outcome variables of our model, has received a lot of attention from empirical studies. Their results depend on the operationalization of the social capital concept but are typically positive, see e.g. Durlauf and Fafchamps (2005) for an overview of these results. In particular, Putnam et al. (1993) and Putnam (2000) observe that bridging social capital (as opposed to bonding social capital) correlates with civil liberties, support for equality and democracy, and low corruption. Similarly Beugelsdijk and Smulders (2003) analyze how bridging and bonding social capital relate to economic growth. There is also ample evidence on influence of bridging contacts on individual economic success (Granovetter, 1973; Podolny and Baron, 1997; Mouw, 2003; Slomczynski et al., 2005;
Franzen and Hangartner, 2006; Growiec and Growiec, 2010; Zhang et al., 2011) while dominance of bonding contacts may have an opposite effect (Franzen and Hangartner, 2006; Sabatini, 2009; Kim, 2009). As regards social utility – non-economic benefits from maintaining social ties – the literature stresses that it is positively correlated to frequent social interactions in general and centrality in the network (see e.g. Winkelmann, 2009; Alesina and Giuliano, 2010; Kroll, 2011; Leung et al., 2011; Growiec and Growiec, 2014; Christakis and Fowler, 2009).

2.3. Empirical evidence from a survey on social capital dimensions

Growiec et al. (2017) present results of a survey of a representative sample of the Polish population, providing direct evidence on the correlations discussed above. Namely they quantify the relationships between:

(i) degree, centrality, bonding and bridging social capital,
(ii) trust and willingness to cooperate,
(iii) social utility and economic performance

on an individual level. We use these results as a basis of model setup and parametrization.

The key empirical findings of Growiec et al. (2017) are the following:

(i) individual's degree positively correlates with centrality and bridging social capital, and negatively with bonding social capital,
(ii) centrality correlates positively with bridging and negatively with bonding social capital,
(iii) bridging and bonding social capital are essentially uncorrelated (this emphasizes our earlier discussion justifying treatment of these variables as independent dimensions).

We want our model to capture all the relationships described above (in Section 4 we show that this postulate is met emergently as we do not directly calibrate model against these correlations).

Additionally, Growiec et al. (2017) study, in line with the literature discussed above, confirms existence of the following correlations:

(i) positive link between bridging social capital and willingness to cooperate,
(ii) positive link between social trust and willingness to cooperate,
negative link between bonding social capital and social trust,
and consequently we incorporate these findings in the assumptions of our model.

3. Specification of the model

The laboratory for our experiments is a computational multi-agent model with a realistic
demographic and social network structure and dynamics. The setup of this model has been
motivated by background literature and individual-level empirical evidence for the Polish population,
presented in Growiec et al. (2017). It extends a related model with a static network structure,
developed in Growiec et al. (2018).

We assume that we have \( N \) agents in the model. The model is dynamic with discrete time
indexed by natural numbers.

3.1. Attributes of agents

Each agent has three attributes: \( f_i \) (family location), \( v_i \) (agent type) and \( a_i \) (age).

3.1.1. Family location

Family location of agent \( i \) is denoted as \( f_i \in [0,1] \). For two agents \( i \) and \( j \) the smaller the
difference between \( f_i \) and \( f_j \) is, the closer are the family ties between them. We want to treat every
value of \( f_i \) in the same way (the value itself should not carry any information, only the difference
between two values should be important) therefore we assume that values 0 and 1 are considered
identical, i.e. we assume that agents are located on a circle. Following this we define a family
similarity \( s_f \) between agents \( i \) and \( j \) as:

\[
s_f(i,j) = 1 - 2 \min\{|f_i - f_j|, 1 - |f_i - f_j|\}.
\]

To calculate \( f_i \) we draw it from a uniform distribution over the interval \([0,1]\), and thus agents
are also uniformly distributed on a circle.

3.1.2. Agent type

Type of agent \( i \) is denoted as \( v_i \in R \) and represents its individual characteristics such as
gender, education, skills etc. For two agents \( i \) and \( j \), the smaller the difference between \( v_i \) and \( v_j \) is,
the more similar are their characteristics. We assume that values of \( v_i \) are normally distributed and can be more or less typical: values close to 0 are considered typical, whereas values that are very positive or very negative are non-standard. We do not impose any single interpretation of types and simply restrict ourselves to considering if some type is typical for the population or not. However, we assume that less typical agents (far from 0) offer potentially more unique values to their connections so they would tend to be more central in the network. Therefore, we define a type distance \( d_v \) between agents \( i \) and \( j \) as:

\[
    d_v(i, j) = 1 - \exp(-|v_i - v_j|).
\]

For each agent we randomly simulate \( v_i \) from a standard normal distribution, as this is a natural distribution to assume for a trait in the population.

### 3.1.3. Agent age

The model has an overlapping generations structure. Each agent has age \( a_i > 0 \) measured in years, where \( a_i = 0 \) is the moment of birth of a person.

There is a probability \( z(a_i) \) that agent of age \( a_i \) dies in the given period, where the simulation tick size (i.e. going from time \( t \) to \( t + 1 \)) is one year. Following Boucekkine et al. (2002) and Azomahou et al. (2009) we posit that the unconditional probability of survival until age \( a_i \) is equal to:

\[
    m(a_i) = \frac{e^{-\kappa a_i - \eta}}{1 - \eta}, \quad \eta > 1, \kappa < 0.
\]

The maximum lifetime of an individual is given under this survival law by \( T^* = -\frac{\ln \eta}{\kappa} \), whereas individuals’ life expectancy is equal to \( E = \frac{1}{\kappa} + \frac{\eta \ln \eta}{(1 - \eta) \kappa} \). For example, when \( \eta = 5.44 \) and \( \kappa = -0.014729 \) as in Boucekkine et al. (2002) then the life expectancy is \( E = 73 \) years and the maximum lifespan is \( T^* = 115 \) years.

We assume that there is a constant birth rate in the social system, equal to the reciprocal of the life expectancy, \( b = 1/E \). This guarantees that the expected value of overall population size is fixed.
The initial ages $a_i$ for the simulation are drawn from the ergodic agent age distribution implied by the function $z(\cdot)$. Coupled with the assumption of a fixed overall population size, this implies that in the ergodic agent age distribution, the number of individuals aged $a_i$ is proportional to $z(a_i)$.

### 3.2 Initial graph of connections

We consider a dynamic population and an evolving graph of connections. Therefore we need to specify the initial structure of the network at the start of the simulation (the population will evolve away from it in the long term but we need to specify a starting point).

Agents at age $a_i < T^*$ do not have connections in the model as they are assumed to be in their school period and are not economically active (they get their initial connections when they reach $a_i = T^*$). We assume that agents enter the market at age $T^* = 15$. To initiate the model at time zero we generate the network of connections using the Watts-Strogatz (Watts and Strogatz, 1998) model over agents that initially have $a_i \geq T^*$. This model has three parameters: $N$ denoting the number of agents in the model, $r$ denoting the graph radius ($2r$ is the average degree of node in the social graph) and $p$ denoting the edge rewiring probability (inverse probability of occurrence of local cliques). In this text we adapt the standard Watts-Strogatz algorithm by assuming that initially agent $i$ is connected to agents $2r$ agents that have the smallest distance $s_f$ to her (in the standard Watts-Strogatz model agents are placed uniformly on a ring lattice, here the distribution of $s_f$ does not have to be uniform). Next with probability $p$ each existing link is replaced by a random link. Hence, the resulting initial graph is always between a lattice ($p = 0$) and a random network ($p = 1$).

It should be stressed that the Watts-Strogatz algorithm does not ensure right skewness of the degree distribution of the graph (a phenomenon observed empirically). However, as it is discussed later, in the long run our simulation exhibits this desirable property, i.e. a heavy right tail of the vertex degree distribution in the social ties graph is an emergent property of our model. Therefore, we emphasize that the Watts-Strogatz model is used only to initiate the simulation and
the steady-state network of connections between agents is different (and better fitting empirical evidence). Actually, we believe that this is a crucial emergent feature of our model.

3.3 Model dynamics

By $x_{i,j}^t \in \{0,1\}$ we denote if in period $t$ agents $i$ and $j$ had a connection. Connections generated by an initial graph are given in period 0. By $L_{i,j}^t$ we denote the shortest path between agents $i$ and $j$ in the graph. If there is no path between $i$ and $j$, it is assumed that $L_{i,j}^t = N_t$, where $N_t$ is the number of agents in period $t$ after the death-birth process described below.

A single step of simulation is interpreted as one year of real time. Connection dynamics are governed by the following rules in a single step of the simulation:

1) death process: we traverse agents in random order; with probability $z(a_i)$ agent $i$ dies (and her social ties are destroyed);

2) birth process: we spawn new agents with age $a_i = 0$ so that in the long run the community size stays unchanged;

3) for each living agent $a_i$ is increased by 1.

4) entry: for every agent who has $a_i = T$, we first generate $2r$ connections with closest family $f_i$ and next rewire each connection with probability $p$ (all actions are only performed against agents that have $a_j \geq T$); in general this step resembles the initial graph generation process described above but is applied only to a single new vertex in the graph;

5) destruction of social ties: all edges are traversed in random order; with probability proportional to $p_{\text{cut}}$ edge from agent $i$ to agent $j$ is cut; $p_{\text{cut}}$ depends on the age of agents $i$ and $j$ (the lower the age the higher the probability) and on their family distance (it is harder to cut family ties);

6) creation of new social ties: all agents are traversed in random order; with probability proportional to $p_{\text{add}}$ agent $i$ creates a new connection; $p_{\text{add}}$ positively depends on agent's degree and negatively depends on her age; next the agent randomly chooses the new connection but the probability weight $p_{\text{con}}$ of connecting with agent $j$ is negatively correlated
with $L_{i,j}$ (i.e. it is more probable to connect with a friend of a friend) and positively with the increase in social utility and economic performance generated by this new connection.

### 3.3.1. Social tie destruction process

In one year $\alpha N_{t}^{*}/2$ ties are destroyed, where $N_{t}^{*}$ is the number of agents in the social graph (with age at least $T^{*}$) and $\alpha$ is a model parameter. We select the set of dissolved ties as a weighted sample from the set of all edges in the graph without replacement. The weight formula for cutting the edge between agents $i$ and $j$ is:

$$p_{cut}(i,j) = \exp\left(-\frac{\min\{a_i, a_j\} - T^{*}}{T^{*}}\right)\left(1 - s_f(i,j)\right)^\beta, \quad \beta > 0.$$ 

The selection of age and family ties as parameters for social tie destruction follow Growiec et al. (2017), whose analysis of empirical data shows that younger people have a higher tendency to sever relationships, that family ties are more stable, and that those are the two most important factors governing this process.

### 3.3.2 Social tie creation process

In one year $\alpha N_{t}^{*}/2$ ties are created. To this end we first select a set of individuals as a weighted sample from the set of all agents in the graph with replacement. The weight formula for adding the edge for agent $i$ is:

$$p_{add}(i) = \exp\left(-\frac{a_i - T^{*}}{T^{*}}\right)(1 + D_i)^\gamma, \quad \gamma > 0.$$ 

Again, the selection of age and degree as parameters for social tie destruction again follows Growiec et al. (2017), whose analysis of empirical data shows that younger people have a higher tendency to create relationships and that people who already have many connections have a higher probability of creating new ones (preferential attachment).

Second, we posit that when an agent decides to create a new social tie she wants to increase her social utility and economic performance. Specifically, for each connection that agent $i$ is about to create she randomly selects agent $j \neq i$ with probability proportional to:
\[ p_{\text{con}}(i, j) = \left( \omega \left( \frac{\Delta SU_i(j)}{\Delta SU_i} \right)^\phi + (1 - \omega) \left( \frac{\Delta EU_i(j)}{\Delta EU_i} \right)^\phi \right)^{\frac{1}{\phi}}, \quad \omega, \phi, \zeta \in (0,1), \]

where \( \Delta SU_i(j) \) and \( \Delta EU_i(j) \) are defined as approximations of the change in agent \( i \)'s social utility \( SU_i \) and economic performance \( EU_i \) that would be produced by connecting to agent \( j \). \( \Delta SU_i \) and \( \Delta EU_i \) are their respective means across all \( j \). The definitions of \( SU_i \) and \( EU_i \) are exactly as in our earlier model with a static social network, Growiec et al. (2018); a detailed justification and an explanation of all relevant parameters is given there. Here we only provide the key elements of the model needed to understand its logic:

\[ \Delta SU_i(j) = s_f(i,j)^p Q_i^{1-p}, \]

where:

\[ Q_i = \frac{\left( \lim_{x \to c_i} F_{\text{emp}}(x) + \lim_{x \to c_i} F_{\text{emp}}(x) \right)}{2}, \]

\( C_i \) is eigenvector centrality of agent \( i \), and \( F_{\text{emp}} \) is the empirical cumulative distribution function of \( C_i \) in population. In other words, \( Q_i \) is the rank of agent \( i \) in terms of her centrality in the graph and \( \Delta SU_i(j) \) is a weighted geometric average of family closeness of \( i \) and \( j \) and social position of agent \( j \).

For economic performance \( \Delta EU_i(j) \) we first define (this follows the assumption that agents are randomly matched and play a stochastic “prisoner’s dilemma game” that is introduced in Section 2 and explained in detail in Growiec et al. (2018)):

\[ EU_i(j) = P_{i,j} d_v(i, j) (W_{i,j} W_{j,i}(\varepsilon^2 + \varepsilon(1 - \varepsilon)(g_{cn} + g_{nc}) - \varepsilon(2 - \varepsilon)g_{nn}) + g_{nn}), \]

where:

\[ W_{i,j} = B_{t_i}/L_{i,j}, \]

is agent \( i \)'s willingness to cooperate with agent \( j \),

\[ P_{i,j} = \frac{\sqrt{(1 - B_{o_i})(1 - B_{o_j})}}{L_{i,j}} \]
is agent i’s trust towards agent j, determining the probability of any economic interaction between
them,
\[ B_{oi} = \begin{cases} \sum_{j:x_{i,j}=1} s_f(i,j)/D_i & \text{if } D_i > 0, \\ 0 & \text{if } D_i = 0 \end{cases}, \]
is bonding social capital of agent i and
\[ Br_i = \begin{cases} \sum_{j:x_{i,j}=1} d_v(i,j)/D_i & \text{if } D_i > 0, \\ 0 & \text{if } D_i = 0 \end{cases}, \]
is her bridging social capital.

The interpretation of the above formulas is that the economic value of a relationship
between agent i and j is proportional to the probability that they get in contact \( P_{i,j} \) (which depends
on their distance in the graph and bonding social capital – two variables that are identified as
important determinants of social trust in the literature review) and to the potential value of this
cooperation \( d_v(i,j) \) that is corrected by assuming that i and j play a stochastic “prisoner’s dilemma”
game (and so they can defect if their willingness to cooperate \( W_{i,j} \) is low; while individually
beneficial, defection decreases the aggregate economic outcome of the cooperation). Willingness to
cooperate \( W_{i,j} \), again following the literature, is related to agents’ bridging social capital and the
distance between agents.

We calculate \( \Delta EU_i(j) \) as the difference between \( EU_i(j) \) when there is no connection
between i and j (and thus \( L_{i,j} > 1 \)) and when the connection is formed (\( L_{i,j} = 1 \)) without updating
all properties of the graph.

A key model design decision was that we assume that social utility \( SU_i \) and economic
performance \( EU_i \) do not depend directly on agent’s age. This assumption is clearly not valid in real
life, as e.g. people accumulate wealth and work experience in the course of their lives, earnings
typically exhibit a hump-shaped profile over individuals’ life cycle, whereas happiness is U-shaped.
However, our objective was to focus on network-induced effects and do not mix them with age.
effects. Additionally, observe that agent's decision making rules depend on $\Delta SU_i(j)$ and $\Delta EU_i(j)$, which would net-out the age effect anyway.

4. Simulation experiment design and results

4.1. Baseline parameterization of the model

The proposed model is computing intensive as it requires simulation of the whole population with complex decision making rules about social tie creation and destruction. Therefore, in order to attain high performance of the code for a reasonably large population, the simulation was developed in Julia language (Bezanson et al., 2017) using LightGraphs package.

Baseline model parameters are given in Table 1. They take into account empirical data and background literature in the following way. Parameters $\kappa$ and $\eta$ were specified following Boucekkine et al. (2002). Value of $N$ was determined by technical reasons – so that the model would fit in computer memory and simulations would be finished in acceptable time. Parameter $r$ was selected based on works of Dunbar and Spoor (1995) and Hill and Dunbar (2003). Parameter $p$ should be small – we assume that when person becomes adult her ties are relatively clustered as they mostly originate from family and school friends. Parameter $\rho$ should be large as confirmed by World Value Survey (http://www.worldvaluessurvey.org/wvs.jsp) data. Relation of parameters of the stochastic “prisoner’s dilemma” game should be $g_{cn} < 0 < g_{nn} < 1 < g_{nc}$, as we take 1 as the reference point where both agents cooperate and 0 means no contact at all. All the assumptions follow the discussion presented in Section 2. Similarly, we conclude that the model parameter $\varepsilon$ should be relatively high. Parameter $\alpha$ follows data from Growiec et al. (2017). Parameter $\beta$ is selected to strongly favor cutting ties with people very weak in family kinship. Parameter $\gamma$ is set to 1 so that the probability of new connections scales linearly with the number of existing ties (if person A has two times more ties than person B then in the past she approximately was twice as active; we assume that she will be also twice as active in the future). Parameter $\omega$ implies that we assume that both economic performance and social utility are equally important for agents. As regards the
substitutability parameter $\phi$, we have tested the model for $\phi < 0$ and the results of the simulation are not in line with empirical data. Therefore, we conclude that for creating new ties, additional social utility and economic performance are substitutable, with an elasticity of substitution above one; we select $\phi = 0.5$. Parameter $\zeta$ is selected to strongly differentiate very short paths from longer ones (i.e. the change of value of $L_{i,j}$ from 2 to 3 is more important than e.g. from 4 to 5, as those are distant ties anyway).

4.2. Results of simulation of the model under its baseline parameterization

The model under its baseline parameterization was simulated 32 times. The results imply that the model replicates reasonably well the correlation structure of empirical data on individuals’ age, degree, centrality, bridging and bonding social capital (Growiec et al., 2017). In Table 2 we report signs of correlations between these variables (they agree between the model and empirical data in every cell in every of 32 runs). We concentrate on signs of relationships as we do not fit the model to quantitatively replicate the empirical data for a single country (Poland) exactly, but want to capture the right direction of the relationship.

In our model the graph of social ties is dynamic and therefore we measure its average properties in the long run. For 1024 agents we observe an average path length in the graph (average of $L_{i,j}$) around 3, global clustering coefficient around 0.27 and the skewness of degree distribution is over 0.5. This means that the assumed graph generating process (edge destruction and creation) leads to small-world type graphs. For reference, in the Watts-Strogatz model with the same number of nodes and edges, calibrated to reach similar average path length we get that: (a) the clustering
coefficient is around two times smaller and (b) skewness is around three times smaller. Thus our model is able to much better reproduce small-world type properties observed for actual social networks.

Here we would like to highlight two facts. First, it is known that Watts-Strogatz graph is not able to adequately represent the long right tail of degree distribution, so the ability to capture this empirically observed phenomenon is a direct advantage of our model. Second, although Watts-Strogatz model was designed to allow for a high clustering coefficient, it is achieved only via a trade-off with average path length. In empirical networks, however, as reported e.g. for Facebook social graph by Ugander et al. (2011), the clustering coefficient is very high despite short path lengths (remaining over 0.2 for nodes with degree less than 20). Therefore, we believe that the fact that our model exhibits a relatively higher clustering coefficient than Watts-Strogatz model can be considered as its relative advantage (it should be noted here, however, that we are not calibrating the model directly to replicate the empirical local clustering coefficient so we concentrate on relative values only).

Finally Table 3 presents factors that influence agent’s economic performance and social utility. They are all in line with expectations. If a sign of correlation is given, it was consistent for all 32 runs of the simulation. In two cells we report 0, which means that the sign of the simulation result was changing and its deviations from 0 were small (we interpret it as no relationship between respective parameters). Again, we concentrate on signs of relationships. We observe that higher degree, centrality and bridging social capital uniformly improve both dimensions of individual performance, while higher bonding improves social utility while decreasing economic performance. People with unique qualities (high $|\nu|$) have higher economic performance, while social utility increases with age. The neutrality of age with respect to economic performance is to be expected as we do not model accumulation of capital in the model, as explained in Section 3.

TABLE 3 around here>
Apart from the correlation analysis we have performed an analysis of goodness of fit of our model against Growiec et al. (2017) data.

In the figures presented in Appendix A we provide a visual comparison of kernel density estimates of key model variables: age (Fig. A-1), the four dimensions of social capital (Fig. A-2), and the four outcome variables (Fig. A-3); as well as nonparametric regressions of these variables against – respectively – individuals’ age (Fig. A-4 and A-5), network degree (Fig. A-6 and A-7), centrality (Fig. A-8), bridging (Fig. A-9) and bonding social capital (Fig. A-10), social trust and willingness to cooperate (Fig. A-11). In general, we find that the model results coincide with the data (although – as we stressed before – it was not directly calibrated to reflect all those relationships). However, a few caveats must be kept in mind when looking at these comparisons:

1. There is no direct measurement of social utility in the data. The variable is computed only as a residual from regressing happiness on incomes, mirroring the model assumption that social utility and economic performance are the two ultimate sources of happiness. In reality, however, happiness may be shaped by other variables as well, such as health, innate optimism, etc. Thus the empirical measurement of this variable is likely noisy.

2. The data on degree (number of acquaintances) are likely to be noisy because the respondents found it hard to recall the number of their social ties. This may artificially reduce the observed correlations between degree and variables such as social trust or willingness to cooperate.

3. In the data we measure bonding social capital as the share of kinship ties among all social ties of an individual. While this corresponds well to the model, it appears that the data on this variable are rather noisy (for the same reason as above), again artificially lowering the correlation of this variable with all others.

4. The model concentrates on the role of cooperation in social networks for economic performance but abstracts from numerous other relevant determinants of individuals’ incomes, such as human capital (education, work experience, health).
5. The model assumes that social utility is drawn from ties with kin and with agents who are central to the network. This a stylized specification which ignores that social utility can be also drawn from social ties with close friends, organized leisure activities, etc.

4.3. Comparative statics analysis of the model

In order to perform sensitivity analysis of our model we have performed a comparative statics experiment consisting in modifying model parameters, one at a time, while keeping other parameters at their baseline levels. This allowed us to identify aggregate and individual-level effects of varying certain characteristics of social capital dynamics.

The ranges for parameter variation in the comparative statics analysis that are reported in this section are provided in Table 1. We report results for all parameters that significantly influence simulation results (we have also tested other parameters and the simulation is either insensitive to changing them within a reasonable range of variation or the results qualitatively do not change).

The results of our study are summarized in Table 4 and discussed below.

<TABLE 4 around here>

4.3.1. Frequency of social network reconfiguration (α).

More frequent social network reconfiguration means that, on average, individuals form more social ties during their lifetime but these ties are less durable. It reduces the role of networks formed in one's youth, which tend to be family- and school-based, as more of them will eventually be replaced with consciously created social ties with people from one's work, hobby groups, etc.

More frequent social network reconfiguration throughout people's lives implies that the resultant network becomes more connected and inclusive, with a reduced average path length and global clustering coefficient. As the percentage share of kinship ties declines, there is also less bonding social capital in the society.
Reduced average path length in conjunction with reduced bonding social capital are conducive to marked increases in social trust and willingness to cooperate. This in turn fuels a strong positive impact on economic performance.

The effects on social utility are \textit{a priori} ambiguous: social utility is drawn from contacts with agents displaying high family similarity as well as highly central individuals, and more frequent network reconfiguration drags the first variable down while pushing the second one up. On balance, the first channel appears stronger and social utility is somewhat reduced.

In summary, a higher $\alpha$ makes the network more similar to a random network: average path length and clustering coefficient decrease, resulting in a higher economic performance at the cost of decreasing social utility. Therefore, we observe that there is no \textit{a priori} optimal value of this parameter in our model as its changes lead to a trade-off between social utility and economic performance (thus extending the results of Growiec et al., 2018, based on a model with a static network structure).

There are also a few interesting results at the individual level. Namely, more frequent network reconfiguration amplifies the cross-sectional correlation between individuals' degree and centrality, while reducing the correlation between these two characteristics and bonding social capital.

\textbf{4.3.2 Importance of family for social tie durability ($\beta$).}

Family ties are more durable than non-family ties (Granovetter, 2005; Roberts, Dunbar, 2011). It is therefore natural to assume that the probability of a tie being dissolved should decline with family similarity. The strength and functional form of this relationship, governed by the parameter $\beta > 0$, are however uncertain. In our simulation experiment, if family similarity is very important for social tie durability ($\beta > 1$) then the relationship is convex and kinship ties are far more difficult to dissolve than non-kinship ties. In contrast, if $\beta < 1$ then the relationship is concave and family similarity only mildly affects the probability of tie destruction.
We find that a high $\beta$ increases the likelihood that the network becomes clustered into local family-based cliques. It therefore increases the global clustering coefficient and the average level of bonding social capital. This, in turn, reduces social trust and economic performance but increases social utility.

In summary, a higher $\beta$ makes the network more clustered around family. This results in lower economic performance and increased social utility, so, similarly to the case of parameter $\alpha$, there is no \textit{a priori} optimal value of this parameter in our model, owing to an underlying trade-off between social utility and economic performance.

At the individual level, we also find that in societies with a higher $\beta$, implying a more clustered social network, individuals' centrality does not correlate that strongly with degree. Also the correlation between both these variables and bonding social capital is reduced.

4.3.3. Frequency of local cliques in adolescents’ social networks ($p$).

Social networks with which adolescents enter their adult life are shaped mostly in family, school and neighborhood. Forming these ties is typically not a conscious choice of the adolescent but a consequence of existing culture and institutions. In terms of our model, a higher $p$ corresponds to more inclusion and less fragmentation into local cliques. Societies with a high $p$ are thus the ones with, on the one hand, less fractionalization, class divisions, and school segregation, and on the other hand, with weaker kinship ties.

We find that more inclusive networks among the youth lead to a less clustered network among the adults as well. They are also conducive to more social trust and better economic performance, but slightly less social utility.

In summary, a higher $p$ makes the network more similar to a random one. This results in higher economic performance and decreased social utility. Again – both performance measures are subject to a trade-off and there is no \textit{a priori} optimal value of this parameter in our model.

At the individual level, more inclusive networks among the youth also imply a higher correlation between agents’ degree or centrality and their bonding social capital.
4.3.4. Other parameters: \( r \) and \( \rho \).

The parameter \( r \) governs the social network density among the youth (\( 2r \) is the average number of social ties per agent when they enter adulthood). It then also relates to the average network density among adults. At face value (Table 4), it has substantial impacts on all model variables. One should be cautious when drawing direct inferences from changes in \( r \), however, because in the simulations this parameter is kept independent from \( \alpha \), the frequency of social network reconfiguration, even though in reality they may be intertwined. Thus a major part of the results may be driven by the implication that more dense networks are also more stable: for a larger \( r \), given a constant \( \alpha \), there is a lower probability a given tie will be dissolved, and newly created ties constitute a lower fraction of all individuals' ties.

The parameter \( \rho \) captures the prevailing social norm on family importance. In societies with a high \( \rho \), family is perceived as relatively important for social utility when compared to social ties outside of family. We find that in societies where \( \rho \) is larger, on average individuals tend to derive more social utility at the cost of lower economic performance. The impact on other variables is, however, rather negligible.

In conclusion, also in this case, for both parameters \( r \) and \( \rho \) we see a tradeoff between economic performance and social utility in our model.

5. Concluding remarks

In this paper we have proposed a computational multi-agent model of dynamic formation of social networks guided by the prospects of improving economic performance and social utility of the agents. The model exhibits high compatibility with empirical data presented in Growiec et al. (2017) and literature review.

The resulting network of connections between agents exhibits small-world properties (small average path length, high clustering and right skewness of the degree distribution). To our knowledge this is the first model that achieves these properties and at the same time: (a) is dynamic (i.e. the network evolves all the time while maintaining these properties and the effect is emergent
endogenously), (b) takes the age structure of the society into account and (c) the process of social tie creation is explicitly guided by both economic performance and social utility maximization motivations.

The findings of comparative statics analysis provide model-based support for a few well-known sociological and psychological theories (e.g., Coleman, 1988; Putnam, 2000; Burt, 2005; Granovetter, 2005). They are also in line with cross-country empirical data to the extent they exist (see the discussion in Growiec et al., 2018). In particular, our result that societies whose social networks are relatively frequently reconfigured, display relatively higher social trust, willingness to cooperate, and economic performance at the cost of lower social utility, correspond with Alesina and Giuliano’s (2010) point that societies with stronger family ties are less trustful but happier. Our finding that more dense networks are also more stable is in agreement with Burt’s (2000, 2002) result that more deeply embedded ties (supplemented by many indirect connections through third parties) are more durable.

As a possible avenue for further research we can identify additional cross-country verification of predictions of our model. In order to perform it, data similar to Growiec et al. (2017) would have to be collected for different populations. Furthermore, one could also extend the model by incorporating e.g. the life-cycle of earnings, human capital accumulation, or the accumulation of wealth.

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Appendix A. Goodness of fit tests of baseline parameterization of the model against empirical data from Growiec et al. (2017).

Figure A-1. Kernel density estimate of the age structure in the population. Data (left panel) vs. model (right panel)
Figure A-2: Kernel density estimates of the structure of social capital variables in the population.

Data (top panel) vs. model (bottom panel)
Figure A-3. Kernel density estimates of the structure of outcome variables in the population. Data (top panel) vs. model (bottom panel)
Figure A-4. Age profiles of social capital variables. Data (top panel) vs. model (bottom panel)
Figure A-5. Age profiles of outcome variables. Data (top panel) vs. model (bottom panel)
Figure A-6. Profiles of social capital variables against agent degree (number of social ties). Data (top panel) vs. model (bottom panel)
Figure A-7. Profiles of outcome variables against agent degree (number of social ties). Data (top panel) vs. model (bottom panel)
Figure A-8. Profiles of outcome variables against agent centrality. Data (top panel) vs. model (bottom panel)
Figure A-9. Profiles of outcome variables against agent's bridging social capital. Data (top panel) vs. model (bottom panel).
Figure A-10. Profiles of outcome variables against agent’s bonding social capital. Data (top panel) vs. model (bottom panel)
Figure A-11. Profiles of outcome variables against social trust and willingness to cooperate. Data (top panel) vs. model (bottom panel)
Figures and tables

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<th>Parameter</th>
<th>Baseline value</th>
<th>Comparative statics range</th>
<th>Interpretation</th>
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</tr>
<tr>
<td>$\eta$</td>
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<td>share of family ties in social utility</td>
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<td></td>
<td>payoff in no-cooperation/cooperation situation</td>
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<tr>
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<td>probability that agent keeps the promise to cooperate</td>
</tr>
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<td>average number of actions of agent per year</td>
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<td>social tie destruction: family importance</td>
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<td>relative weight of social utility and economic perf.</td>
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<td>$\phi$</td>
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<tr>
<td>$\zeta$</td>
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<td>decay rate of distance in network for economic utility</td>
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Table 1. Parameterization of the baseline simulation experiment and comparative statics ranges.

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<th>bridging</th>
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<td>+</td>
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<tr>
<td>bonding</td>
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Table 2. Correlation of base output parameters of simulation experiment; identical to empirical data from Growiec et al. (2017).
Table 3. Factors influencing economic performance and social utility of an agent (we report sign of relationship in a linear regression metamodel, 0 indicates approximately no relationship).

|                      | \( |\nu| \) | age | degree | centrality | bridging | bonding |
|----------------------|----------|-----|--------|------------|----------|---------|
| economic performance | +        | 0   | +      | +          | +        | −       |
| social utility       | 0        | +   | +      | +          | +        | +       |

Table 4. Comparative statics analysis results around the baseline calibration. In the table "—" signifies negative relationship, "+" signifies positive relationship, "+ +" signifies strong positive relationship, 0 signifies very weak relationship and U signifies U-shaped relationship between parameters.

<table>
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<tr>
<th></th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( p )</th>
<th>( r )</th>
<th>( \rho )</th>
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<td>0</td>
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<td>−</td>
<td>+</td>
<td>+ +</td>
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<td>+</td>
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