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Empirical power of the Kwiatkowski-Phillips-Schmidt-Shin test

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Abstract

The aim of this paper is to study properties of the Kwiatkowski-Phillips-Schmidt-Shin test (KPSS test), introduced in Kwiatkowski et al. (1992) paper. The null of the test corresponds to stationarity of a series, the alternative to its nonstationarity. Distribution of the test statistics is nonstandard, asymptotically converges to Brownian bridges as was shown in original paper. The authors produced tables of critical values based on asymptotic approximation. Here we present results of simulation experiment aimed at studying small sample properties of the test and its empirical power.

JEL classification codes: C120, C16

Keywords: KPSS test; stationarity; integration; empirical power of KPSS test

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1. *Introduction*

The aim of this research is to investigate properties of the Kwiatkowski- Phillips-Schmidt-Shin test (henceforth KPSS test) – introduced in 1992, test of stationarity of time series versus alternative of unit root[2].

Unit root tests (starting with classic Dickey-Fuller test, and several refinements, Perron-type tests), have as a null hypothesis presence of unit root in the series. The alternative of stationarity is a joint hypothesis. The KPSS test differs from the majority of tests used for checking integration in that its null of stationarity is a simple hypothesis.

In the first part of this paper we remind definition of the DF tests and behaviour of integrated and stationary series. Second part, based on original Kwiatkowski et al. (1992) paper, describes the KPSS test and its asymptotic properties. In the third part we present results of the simulation experiment, aimed at computation of percentiles of the KPSS test statistic, and investigation of empirical power of the test. Fourth part compares results of application of the DF and KPSS test to several macroeconomic data series. Last part concludes.

Comparison of the results obtained in usual DF framework with KPSS test statistic gives possibility to check whether series is stationary, or is non-stationary due to presence of a unit root, or – as may happen – data do not contain information enough for conclusions. Hence critical values for finite samples and analysis of the empirical power of the KPSS test are so important.

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2 This research was performed during author's stay at Central European Economic Research Center (the financial support of this projest is gratefully acknowledged), on leave from the Warsaw School of Economics, and the first version of paper was published in 1997 as a Working Paper on the CEEERC website. As this website ceased to exist, after checking small deficiencies, the author decided to publish it again.
I am grateful to colleagues from Warsaw University, CEERC, and Warsaw School of Economics for discussions, and to referees for their kind remarks. All remaining deficiencies are mine.

2. *Integration and Dickey-Fuller test*

This section briefly reminds definition of *DF* test and properties of integrated series. Let us assume that series of observations of a certain variable y is generated by an AR(1) process:

(1) $$y_t = \alpha\, y_{t-1} + \varepsilon_t$$

where: ε_t is a stationary disturbance term. If $\alpha = 1$ (i.e., if characteristic equation of the process (1) has a unit root) then the process is nonstationary. As follows from assumption of stationarity of ε_t, first differences of y are stationary. The series $\{y_t\}$ is integrated of the first order, I(1). If $|\alpha| < 1$, then $\{y_t\}$ is stationary in the sense that it is integrated of order zero. Order of integration of $\{y_t\}$ determines its properties, e.g. (see Mills, [1993]):

- If $\{y_t\}$ is integrated of order 0, then:
  - its variance is finite and does not depend on t;
  - disturbance ε_t has only transitory effect on y_t;
  - expected time between crossing of zero is finite, i.e., y_t varies around its expected value, 0;
  - correlation coefficients, ρ_k, diminish with increase of lag k, and the sum of ρ_k is finite.
- If the series y_t is integrated of order 1, and $y_0 = 0$, then:
  - variance of y_t tends to infinity with t;
  - disturbance ε_t has a permanent effect on y_t, because y_t is a sum of all previous values of ε_t;
  - expected time between consecutive crossings of the line $y = 0$ is infinite;
  - correlation coefficients ρ_k tend to infinity with increase of k.

Those features of series of observations for a macroeconomic variable have a marked effect, for example, on the results of policy analysis. Hence testing for integration of a series and taking such features into account in process of building an econometric model are so important. The Dickey-Fuller test (Dickey and Fuller [1979], [1981]) is the test of a null hypothesis that in a model

(2) $$\Delta y_t = \delta\, y_{t-1} + \varepsilon_t$$

(which is equivalent to the model (1) for $\delta = \alpha - 1$) the parameter δ is equal to zero (i.e. variable y_t is generated by an AR(1) process), against alternative $\delta < 0$ (i.e. variable is stationary). Assumption about stationarity of the series y_t here, as in various refinements of this test, is an alternative hypothesis. The test statistics is computed as $t = \hat{\delta} / \hat{\sigma}_\delta$, that is in the way similar to the t-ratio for parameter of a lagged variable, but it has different probability density function.

If computed value exceeds a critical value at chosen significance level, then the null hypothesis about presence of unit root in a series cannot be rejected. If computed value is smaller than the critical value, then we reject null in favour of stationarity of the y_t series. As a right-hand side of (2) contains lagged y_t, in general disturbance terms are correlated; the augmented DF test takes care of this correlation by including on the right-hand side of (2) lagged values of differences of y_t.

It is also possible to include a constant:

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \varepsilon_t$$

– when a series $\{ y_t \}$ is stationary around mean, or a linear trend:

$$y_t = \alpha_0 + \xi t + \alpha y_{t-1} + \varepsilon_t$$

– then for $|\alpha| < 1$ the series $\{ y_t \}$ is stationary around linear trend. For $\alpha = 1$, the process y_t contains a unit root and is non-stationary.

3. *The Kwiatkowski, Phillips, Schmidt and Shin test*

The alternative test introduced in 1992 by Kwiatkowski, Phillips, Schmidt and Shin, and called henceforth the KPSS test, has a null of stationarity of a series around either mean or a linear trend; and the alternative assumes that a series is non-stationary due to presence of a unit root. In this respect it is innovative in comparison with earlier Dickey-Fuller test, or Perron type tests, in which null hypothesis assumes presence of a unit root.

In the *KPSS* model, series of observations is represented as a sum of three components: deterministic trend, a random walk, and a stationary error term. The model has the following form:

$$\begin{aligned} y_t &= \xi t + r_t + \varepsilon_t \\ r_t &= r_{t-1} + u_t \end{aligned} \qquad (3)$$

where y_t, $t = 1, 2, \dots, T$ denotes series of observations of variable of interest, t – deterministic trend, r_t – random walk process, ε_t – error term of the first equation, by assumption is stationary,

u_t denotes an error term of second equation, and by assumption is a series of identically distributed independent random variables of expected value equal to zero and constant variation $\hat{\sigma}_u^2$. By assumption, an initial value r_0 of the second equation in (3) is a constant; and it corresponds to an intercept.

The null hypothesis of stationarity is equivalent to the assumption that the variance σ_u^2 of the random walk process r_t in equation (3), equals zero. In case when $\xi = 0$, the null means that y_t is stationary around r_0. If $\xi \neq 0$, then the null means that y_t is stationary around a linear trend. If the variance σ_u^2 is greater than zero, then y_t is non-stationary (as sum of a trend and random walk), due to presence of a unit root.

Subtracting y_t from both sides of the first equation in equation (3) we obtain:

$$\Delta y_t = \xi + u_t + \Delta\varepsilon_t = \xi + w_t$$

where w_t, due to assumption that ε_t, and u_t, are independently identically distributed random variables, is generated by an autoregressive process AR(1) (see Kwiatkowski *et al.* [1992]): $w_t = v_t + \theta\, v_{t-1}$. Hence the KPSS model may be expressed in the following form:

$$\begin{aligned} y_t &= \xi + \beta\, y_{t-1} + w_t, \\ w_t &= v_t + \theta\, v_{t-1},\ \beta = 1 \end{aligned}$$

This equation expresses an interesting relationship between KPSS test and DF test, as DF test checks $\beta = 1$ on assumption that $\theta = 0$; where θ is a nuisance parameter. Kwiatkowski *et al*. assume that β is a nuisance parameter, and test whether $\theta = -1$, assuming that $\beta = 0$. They introduce one-side Lagrange Multiplier test of null hypothesis $\sigma_u^2 = 0$ with assumption that u_t have a normal distribution and ε_t are identically distributed independent random variables with zero expected value and a constant variance σ_ε^2.

The KPSS test statistics is defined in a following way.

A. For testing a null of stationarity around a linear trend versus alternative of presence of a unit root:

Let e_t, $t = 1,2,3,\ldots,T$ denote estimated errors from a regression of y_t on a constant and time. Let $\hat{\sigma}_t^2$ denote estimate of variance, equal to a sum of error squares divided by number of observations *T.* The partial sums of errors are computed as:

$$S_t = \sum_{i=1}^{t} e_i \text{, for } t = 1,2,...,T.$$

The LM test statistic is defined as:

(4) $$\text{LM} = \sum_{t=1}^{T} S_t^2 \,/\, \sigma_\varepsilon^2$$

B. <u>For testing a null hypothesis of stationarity around mean, versus alternative of presence of a unit root:</u> The estimated errors e_t are computed as residuals of regression of y_t on a constant (i.e. $e_t = y_t - \bar{y}$), the rest of definitions are unchanged.

Inference of asymptotic properties of the statistic is based on assumption that ε_t have certain regularity properties defined by Phillips and Perron (1988, p. 336). The long-run variance is defined as:

(5) $$\sigma^2 = \lim T^{-1} E[S_T^2]$$

The long-run variance appears in equations defining asymptotic distribution of a test statistic. The consistent estimate of the long-run variance is given by a formula (see Kwiatkowski *et al.*, [1992]:

(6) $$s^2(k) = T^{-1} \sum_{t=1}^{T} e_t^2 + 2T^{-1} \sum_{j=1}^{k} w(j,k) \sum_{t=s+1}^{T} e_t e_{t-1}$$

where $w(j,k)$ denote weights, depending on a choice of spectral window. The authors use the Bartlett window, i.e. $w(j,k) = 1 - \frac{j}{k+1}$, which ensures that $s^2(k)$ is non-negative. They argue that for quarterly data lag $k = 8$ is the best choice (if $k<8$, size of test is distorted, if $k>8$, power decreases, see Kwiatkowski *et al*. [1992]). The KPSS test statistic is computed as a ratio of sum of squared partial sums, and estimate of long-term variance, i.e. :

$$\hat{\eta} = T^{-2} \sum S_t^2 / s^2(k)$$

Symbols $\hat{\eta}_\mu$ and $\hat{\eta}_\tau$ denote respectively the KPSS statistic for testing stationarity around mean and around a trend.

Asymptotic distribution of the KPSS test statistic is non-standard, it converges to a Brownian bridges of higher order (see Kwiatkowski *et al*. 1992, p. 161). The $\hat{\eta}_\mu$ statistic for testing stationarity around mean converges to:

$$\hat{\eta}_\mu \to \int_0^1 V(r)^2 \; dr$$

where $V(r) = W(r) - r\,W(1)$ denotes a standard Brownian bridge, defined for a standard Wiener process $W(r)$, and $\to$ is weak convergence of probability measures.

The KPSS test statistic $\hat{\eta}_\tau$ for stationarity around trend, i.e. for $\xi \neq 0$, weakly converges to a second order Brownian bridge, $V_2(r)$, defined as

$$V_2(r)_2 = W(r) + \left(2r - 3r^2\right)W(1) + \left(-6r + 6r^2\right)\int_0^1 W(s)\,ds$$

(See Kwiatkowski *et al.* [1992]).

The statistic weakly converges to a limit

$$\hat{\eta}_\tau \to \int_0^1 V_2(r)^2 \; dr$$

The KPSS test is performed in a following way: We test null hypothesis about stationarity around trend, or around mean, against alternative of nonstationarity of a series due to presence of a unit root. We compute value of a test statistic, $\hat{\eta}_\mu$ o r $\hat{\eta}_\tau$, respectively. If computed value is greater than critical value, the null hypothesis of stationarity is rejected at given level of significance.

4. *Critical values of the KPSS test*

In the original Kwiatkowski *et al*. (1992) paper the results of Monte Carlo simulation concerning size and power of the KPSS test and asymptotic properties of the test statistics were obtain with use of equations (9) and (10), which means that the critical values given there are asymptotic. Hence the need of computing critical values for finite sample size.

In what follows I present results of Monte Carlo experiment aimed at computation of critical values for the KPSS test, based on definition (8).

I have used procedure in GAUSS written by David Rapach (address: *http:// netec.mcc.ac.uk/ ~adnetec/CodEc/ GaussAtAmericanU/GAUSSIDX/HTML*).

Data generating process used for simulation corresponds to the model (5) and (6). Number of lags equals 8. The model has the following form:

$$y_t = \xi\ t + r_0 + \varepsilon_t$$

and two versions: for $\xi = 0$ model has a constant only, and for $\xi \neq 0$ – constant and a linear trend.

The test statistic was computed for k=8 as:

$$LM = \sum_{t=1}^{T} S_t^2 \,/\, s^2(8)$$

where:

$$s^2(8) = T^{-1}\sum_{t=1}^{T} e_t^2 + 2T^{-1}\sum_{j=1}^{8} w(j,8) \sum_{t=s+1}^{T} e_t \quad e_{t-j} \,.$$

Sample size was set at 15, 20, 25, 30, 40, 50 60, 70, 80, 90 and 100.

Number of replication equals 50000. The computed critical values of the KPSS test statistic are given in Table 1.

5. *Empirical power of the KPSS test*

Assumptions of a simulation experiment aimed at checking power of the KPSS test were the following. Sample size was set at T=15,20, 25,30, 40, 50, 60, 70, 80, 90 and 100, number of replications was equal to 10000. Data generating process containing a random walk with *non-zero variance* of the error term corresponds to the alternative of the KPSS test, ie., non-stationarity of a series due to presence of a unit root. The error term *variance equal to zero* corresponds to a null hypothesis of stationarity. Earlier experiments have shown that particular value of variance, as long as it was non-zero, had little effect on the results. I assume here that variance takes three values: 0 (as a benchmark), 0.5, 1.0 and 1.5.

Hence data generating process has the following form:

$$y_t = \xi\ t + r_t + \varepsilon_t$$
$$r_t = r_{t-1} + u_t$$

where disturbances ε_t were generated as independent identically distributed variables with normal standard distribution, and u_t – as independent identically distributed random variables with normal distribution. Disturbances of these two equations were mutually independent.

The experiment has been performed for two versions of the DGP: with linear trend and without linear trend. In former case $\xi = 0.1$, in latter case $\xi = 0$. Computed test statistic were compared with the critical values. The results are shown in Table 2.

Table 3 shows the results of checking whether the value of $\hat{\sigma}_u^2$ chosen in simulation has an effect on the empirical power of the KPSS test. The regression was run of a percentage of rejection on two variables: $\hat{\sigma}_u^2 = \{0.0, 0.1, 0.2, ..., 1.4\}$ and $\alpha \in \{0.95, 0.90, 0.50, 0.10\}$. The choice of $\hat{\sigma}_u^2$ does not influence the empirical power of the test for a model with a linear trend. The evidence for model without trend is mixed.

Table 4 presents results of computation of the empirical power of the KPSS test for 25, 30, 40, 50, … 90, 100 observations. In the DGP the variance takes the values: 0 (as a benchmark; this corresponds to a null of stationarity); 0.1, 0.2, … 1.4.

6. *Example: comparison of the DF and KPSS tests for several macroeconomic time series*

In paper written by Dickey *et al.* (1991), reprinted in extended form in book by Rao (1995), the authors show results concerning integration and cointegration of several macroeconomic variables. The data set has been reprinted in the Rao book, it consists of quarterly observations, starting in first quarter of 1953, ending in last quarter of 1988, i.e. covers 36 years – and 144 observations.As usual, testing of integration was an introductory step leading to estimation of cointegration relationship. It was performed with use od the Dickey-Fuller test with three augmentations.

I have repeated the testing for integration using DF test, and applied the KPSS test to the same data, with use of GAUSS 3.2.14 computing package.

The results for the DF test are given in Table 4. They are in perfect agreement with original results of Dickey (1991): the null hypothesis of presence of a unit root cannot be rejected.

My results for the KPSS test are given in Table 4. The symbol # means that computed value of the KPSS test statistic is greater than critical value for 100 observations.

A. Test of stationarity around mean:

For all variables computed KPSS test statistic was greater than the critical value. Hence the null of stationarity around mean is rejected.

B. Test of stationarity around a linear trend:

Only for real money category M1/P and rates of return from 10 Year Government bonds the null of stationarity around a trend cannot be rejected. For all other variables this hypothesis is rejected.

We can conclude that both the DF test and the KPSS test give similar results:

• all variables can be modelled with use of AR model with trend, and

• for money and rate of return from bonds coefficient of autoregression was smaller than 1;

• all other variables have a unit root.

7. *Summary*

My analysis concerning the KPSS test confirms earlier results of Kwiatkowski *et al*. (1992) and later results of Amano (1992). The test, due to its form and to the way of formulating null and alternative hypotheses, should be used jointly with unit root test, e.g. the DF or augmented DF test. Comparison of results of the KPSS test with those of unit root test improve quality of inference (see Amano, 1992). Testing both unit root hypothesis and the stationarity hypothesis helps to distinguish the series which appear to be stationary, from those which have a unit root, and those, for which the information contained in the data is not sufficient to confirm whether series is stationary or non-stationary due to presence of a unit root.

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TABLE 1. Critical values of the KPSS test statistics, for 50000 replications

Sample size = 15

α	Without trend	Linear trend
0.990	0.48313288	0.41433477
0.975	0.45183890	0.38740080
0.950	0.42608752	0.36435597
0.900	0.39875209	0.34151076
0.500	0.31307493	0.27000041
0.100	0.24775830	0.22441514
0.050	0.23429514	0.21764786
0.025	0.22542106	0.21314635
0.010	0.21814408	0.20935949

Sample size = 20

α	Without trend	Linear trend
0.990	0.42612535	0.32710900
0.975	0.40672348	0.30130862
0.950	0.38874144	0.27736290
0.900	0.36425871	0.25185147
0.500	0.25352270	0.18687704
0.100	0.17868235	0.16048566
0.050	0.16906971	0.15720065
0.025	0.13643788	0.15498356
0.010	0.15862807	0.15314154

Sample size = 25

α	Without trend	Linear trend
0.990	0.42646756	0.25070640
0.975	0.40531466	0.22643507
0.950	0.38197871	0.20925595
0.900	0.35089080	0.19228778
0.500	0.21829452	0.15145480
0.100	0.14634008	0.12768145
0.050	0.13698973	0.12327038
0.025	0.13060574	0.12031411
0.010	0.12464071	0.11733054

Sample size = 30

α	Without trend	Linear trend
0.990	0.44132930	0.200256350
0.975	0.41341759	0.182619190
0.950	0.38597981	0.170824210
0.900	0.34684355	0.159814950
0.500	0.19372352	0.130842290
0.100	0.12651386	0.107294630
0.050	0.11659583	0.102870090
0.025	0.10982029	0.099837474
0.010	0.10324302	0.096860084

Sample size = 40

α	Without trend	Linear trend
0.990	0.475156690	0.160712240
0.975	0.433981310	0.153045430
0.950	0.395416130	0.145912950
0.900	0.344645180	0.137426680
0.500	0.169521970	0.105376760
0.100	0.102161600	0.084344769
0.050	0.093148798	0.079845616
0.025	0.086988805	0.076609429
0.010	0.081393647	0.073462161

Sample size = 50

α	Without trend	Linear trend
0.990	0.502280620	0.159601370
0.975	0.452679270	0.149952080
0.950	0.404525140	0.140362660
0.900	0.342136350	0.129087330
0.500	0.156169680	0.091700908
0.100	0.088497158	0.070397237
0.050	0.079463830	0.066548551
0.025	0.073762159	0.063667488
0.010	0.068490918	0.060947315

Sample size = 60

α	Without trend	Linear trend
0.990	0.528078950	0.162823760
0.975	0.468351880	0.150524540
0.950	0.412710640	0.139364470
0.900	0.345339490	0.125373750
0.500	0.150605630	0.083937329
0.100	0.080174225	0.062058256
0.050	0.071325513	0.058300785
0.025	0.065485408	0.055667655
0.010	0.060465037	0.052867329

Sample size = 70

α	Without trend	Linear trend
0.990	0.549813740	0.165854940
0.975	0.480306630	0.151748190
0.950	0.417688110	0.138643110
0.900	0.344672600	0.123379830
0.500	0.144216790	0.078741199
0.100	0.074349269	0.056075523
0.050	0.065356429	0.052371944
0.025	0.059376144	0.049654444
0.010	0.054097437	0.046988381

Sample size = 80

α	Without trend	Linear trend
0.990	0.569931730	0.171982270
0.975	0.493300620	0.154278010
0.950	0.424566150	0.139022010
0.900	0.346647950	0.121706290
0.500	0.141357440	0.075160709
0.100	0.069921053	0.051766687
0.050	0.060900214	0.047949210
0.025	0.054953008	0.045218561
0.010	0.049560220	0.042535417

Sample size = 90

α	Without trend	Linear trend
0.990	0.587101070	0.175304920
0.975	0.505673320	0.156321150
0.950	0.429490220	0.139885170
0.900	0.344908300	0.121399040
0.500	0.139052120	0.072447229
0.100	0.067168859	0.048548798
0.050	0.057816010	0.044612097
0.025	0.051877068	0.041908084
0.010	0.046780441	0.039386823

Sample size = 100

α	Without trend	Linear trend
0.990	0.594603380	0.177754650
0.975	0.510372830	0.157183470
0.950	0.431164860	0.139652320
0.900	0.343732070	0.120403750
0.500	0.135927460	0.070300302
0.100	0.064217752	0.045987879
0.050	0.055225906	0.042096564
0.025	0.049190208	0.039409235
0.010	0.043797820	0.036908227

Table 2. The empirical power of the KPSS test

A. Results for model without trend.
The tested null hypothesis is of level stationarity.

Sample size = 80

Variance	Significance level					
	0.99	0.95	0.90	0.10	0.05	0.01
0.0	0.00850	0.04900	0.09710	0.89570	0.94740	0.99050
0.1	0.01090	0.05100	0.09890	0.90010	0.94900	0.98890
0.2	0.01060	0.05440	0.10530	0.90410	0.95200	0.99030
0.3	0.00980	0.04710	0.09900	0.89800	0.95000	0.99040
0.4	0.01030	0.04760	0.09980	0.90220	0.95170	0.99000
0.5	0.01090	0.05050	0.10010	0.89960	0.95000	0.99000
0.6	0.01050	0.04970	0.09770	0.89900	0.95060	0.99230
0.7	0.01050	0.04930	0.09810	0.89900	0.94560	0.98940
0.8	0.01200	0.04970	0.09630	0.90280	0.95020	0.99050
0.9	0.00930	0.04780	0.09750	0.90510	0.95080	0.99030
1.0	0.08700	0.04820	0.09800	0.89690	0.94880	0.99140
1.1	0.00780	0.04850	0.09910	0.90380	0.95290	0.99100
1.2	0.00890	0.04690	0.09680	0.90190	0.95220	0.99000
1.3	0.00960	0.04610	0.09500	0.89630	0.94850	0.99210
1.4	0.01130	0.04860	0.09580	0.89610	0.94700	0.98940

Sample size = 90

Variance	Significance level					
	0.99	0.95	0.90	0.10	0.05	0.01
0.0	0.00890	0.05060	0.10100	0.89820	0.94910	0.98850
0.1	0.00910	0.04920	0.10110	0.89980	0.94960	0.99070
0.2	0.01200	0.04730	0.09770	0.90270	0.95180	0.99130
0.3	0.01100	0.05180	0.09800	0.90460	0.95370	0.99020
0.4	0.01090	0.05110	0.10230	0.89470	0.94580	0.98940
0.5	0.00960	0.04760	0.09890	0.89760	0.95130	0.99080
0.6	0.00960	0.04740	0.09800	0.90050	0.95370	0.99160
0.7	0.01020	0.04770	0.10340	0.89730	0.94860	0.98930
0.8	0.00990	0.04930	0.09940	0.90080	0.94760	0.98880
0.9	0.00970	0.04650	0.09830	0.89780	0.94720	0.98840
1.0	0.00850	0.04580	0.09850	0.89650	0.94760	0.99100
1.1	0.00910	0.04660	0.09860	0.89680	0.94840	0.98910
1.2	0.01100	0.04830	0.10180	0.90040	0.95150	0.99090
1.3	0.01090	0.04760	0.09450	0.89830	0.94660	0.98830
1.4	0.00730	0.04690	0.09910	0.89640	0.94940	0.98900

Sample size = 100

Variance	Significance level					
	0.99	0.95	0.90	0.10	0.05	0.01
0.0	0.00960	0.04880	0.10480	0.90180	0.95040	0.99160
0.1	0.01050	0.04890	0.09820	0.90420	0.95170	0.99020
0.2	0.00950	0.05190	0.10550	0.89960	0.94860	0.99040
0.3	0.01260	0.05310	0.10560	0.90580	0.95360	0.99060
0.4	0.01060	0.05310	0.10170	0.90810	0.95380	0.99250
0.5	0.00940	0.05000	0.09970	0.89960	0.94960	0.99100
0.6	0.01030	0.04760	0.10010	0.90620	0.95080	0.99120
0.7	0.00850	0.08600	0.09990	0.899930	0.94840	0.99020
0.8	0.01020	0.04780	0.09950	089340	0.94730	0.99040
0.9	0.01170	0.04980	0.10120	090150	0.95240	0.98930
1.0	0.00900	0.04530	0.09410	0.90330	0.95260	0.99120
1.1	0.01010	0.05000	0.10160	0.89680	0.94880	0.98970
1.2	0.00990	0.04860	0.09520	0.90170	0.94810	0.98870
1.3	0.00950	0.04970	0.09970	0.89060	0.94560	0.99040
1.4	0.01130	0.05220	0.10060	0.90070	0.94830	0.99000

B. Results for model with linear trend.

The tested null hypothesis is of stationarity around linear trend

Sample size = 80

Variance	Significance level					
	0.99	0.95	0.90	0.10	0.05	0.01
0.0	0.01020	0.04790	0.09780	0.89440	0.94720	0.99040
0.1	0.00890	0.05220	0.10610	0.89910	0.94910	0.99080
0.2	0.01000	0.04930	0.09960	0.90100	0.95150	0.99030
0.3	0.0960	0.04800	0.09710	0.90220	0.95310	0.98990
0.4	0.01130	0.05260	0.09940	0.90330	0.94910	0.98860
0.5	0.00990	0.5050	0.10160	0.90340	0.95260	0.99050
0.6	0.00930	0.05100	0.10310	0.89920	0.94920	0.99030
0.7	0.01120	0.05240	0.10010	0.89430	0.94680	0.98810
0.8	0.00850	0.04800	0.09970	0.89930	0.94980	0.99180
0.9	0.01090	0.05190	0.10340	0.89910	0.94700	0.99120
1.0	0.00940	0.04940	0.09720	0.90050	0.95170	0.99020
1.1	0.00960	0.05350	0.10340	0.90190	0.95070	0.98920
1.2	0.00960	0.05070	0.10390	0.89810	0.94970	0.98970
1.3	0.00830	0.04810	0.09490	0.90000	0.95260	0.99040
1.4	0.00860	0.04860	0.09730	0.89830	0.94790	0.99070

Source: own computations

Sample size = 90

Variance	Significance level					
	0.99	0.95	0.90	0.10	0.05	0.01
0.0	0.00910	0.04930	0.09630	0.90010	0.95180	0.99080
0.1	0.00940	0.05090	0.09800	0.89990	0.95210	0.99020
0.2	0.00850	0.05020	0.09980	0.89390	0.94740	0.98890
0.3	0.00870	0.04720	0.09700	0.89960	0.94960	0.98960
0.4	0.00860	0.04740	0.09980	0.90020	0.95240	0.99030
0.5	0.01020	0.05020	0.10280	0.89870	0.94940	0.98880
0.6	0.00940	0.04640	0.09770	0.89910	0.95180	0.99040
0.7	0.00920	0.05030	0.10120	0.90380	0.95090	0.99000
0.8	0.00920	0.05170	0.10620	0.89830	0.95030	0.99120
0.9	0.01060	0.04790	0.09860	0.89720	0.94430	0.98850
1.0	0.00950	0.05000	0.10070	0.89820	0.95080	0.98950
1.1	0.01060	0.04930	0.10050	0.89940	0.94890	0.98980
1.2	0.01020	0.05210	010230	0.90090	0.94820	0.98920
1.3	0.01010	0.04470	0.09410	0.89860	0.94850	0.98860
1.4	0.00980	0.04860	0.10040	0.90110	0.95190	0.99090

Source: own computations

Sample size =100

Variance	Significance level					
	0.99	0.95	0.90	0.10	0.05	0.01
0.0	0.00920	0.04970	0.09870	0.89550	0.94960	0.98960
0.1	0.00880	0.04500	0.09830	0.89470	0.94770	0.98770
0.2	0.00690	0.04890	0.09940	0.90070	0.94990	0.98880
0.3	0.00920	0.05200	0.09970	0.90410	095130	0.99030
0.4	0.00850	0.05070	0.10360	0.89930	0.94790	0.98730
0.5	0.01000	0.04530	0.09580	0.90140	0.94910	0.98940
0.6	0.00940	0.05180	010170	0.90060	0.94910	0.98780
0.7	0.01150	0.05170	0.09980	089440	0.94560	0.98910
0.8	0.00920	0.05250	0.10340	0.90580	0.95070	0.98980
0.9	0.01060	0.04950	0.09730	0.89820	0.94920	0.98860
1.0	0.01250	0.05540	010640	0.90600	0.95450	0.98960
1.1	0.00870	0.04780	0.09920	0.90200	0.95060	0.98890
1.2	0.00980	0.05130	0.09950	0.89640	0.94650	0.98770
1.3	0.01100	0.05300	0.10000	0.89970	0.95010	0.99040
1.4	0.00980	0.05190	0.10170	090190	0.95090	0.98960

Source: own computations

Table 3. Effect of choice of $\hat{\sigma}_u^2$ value on empirical power of test

1. For a fixed significance level α of the KPSS test compute its empirical power for different sample sizes and values of $\hat{\sigma}_u^2$.
2. Run a regression of empirical power on sample size and value of $\hat{\sigma}_u^2$.
3. Check significance of $\hat{\sigma}_u^2$ in this regression.

Model without trend	Sample size 80	Sample size 90	Sample size 100
$\alpha =$	The value of $\hat{\sigma}_u^2$ in this regression is:		
0.99	Significant	Significant	Insignificant
0.95	Significant	Significant	Insignificant
0.90	Significant	Insignificant	Significant
0.10	Insignificant	Significant	Significant
0.05	Insignificant	Insignificant	Significant
0.01	Insignificant	Insignificant	Significant

Source: own computations

Model with a linear trend	80 observations	90 observations	100 observations
$\alpha =$	The value of $\hat{\sigma}_u^2$ in this regression is:		
0.99	Insignificant	Significant	Significant
0.95	Insignificant	Insignificant	Significant
0.90	Insignificant	Insignificant	Insignificant
0.10	Insignificant	Insignificant	Insignificant
0.05	Insignificant	Significant	Insignificant
0.01	Insignificant	Insignificant	Insignificant

Source: own computations

Table 4.
The results of the Dickey-Fuller test for macroeconomic variables

M1/P	Real money M1
M2/P	Real money, M2
MB/P	Real monetary base
NM1M2/P	Part of M2 category outside M1, real terms
K	Proportion of cash to checkable deposits
KSA	Proportion of cash to checkable deposits, seasonally adjusted
R3M	Nominal percentage rate for 3-month Treasury Bills
R10Y	Nominal returns from 10-year Government securities
RGNP	Real GNP

Variable	Model with a constant	Model with a constant and a linear trend	Variable	Model with a constant
K	-0.5490	-2.332	Δ K	-4.223*
M2/P	-0.8040	-4.737	Δ M2/P	-10.15*
M1/P	-0.8001	-1.542	Δ M1/P	-3.639*
MP/P	0.4109	-2.624	Δ MB/P	-3.048*
RGNP	-0.4672	-2.412	Δ RGNP	-6.233*
R3M	-2.324	-3.743	Δ R3M	-6.346*
R10Y	-1.874	-2.447	Δ R10Y	-5.590*
NM1M2/P	-2.156	-1.447	Δ NM1M2/P	-4.029*

Source: own computations

Table 5. The KPSS test statistics for the same variables

Variable	Test with a constant	Test with a trend
K	1.385#	0.2334#
M2/P	1.677#	0.2139#
M1/P	0.5210#	0.1382
RGNP	1.686#	0.2623#
R3M	1.380#	0.1717#
R10Y	1.543#	0.1338
NM1M2/P	1.651#	0.3835#

Source: own computations